Continuous-variable quantum teleportation in bosonic structured environments

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The effects of dynamics of continuous-variable entanglement under the various kinds of environments on quantum teleportation are quantitatively investigated. Only under assumption of the weak system-reservoir interaction, the evolution of teleportation fidelity is analytically derived and is numerically plotted in terms of environment parameters including reservoir temperature and its spectral density, without Markovian and rotating wave approximations. We find that the fidelity of teleportation is a monotonically decreasing function for Markovian interaction in Ohmic-like environments, while it oscillates for non-Markovian ones. According to the dynamical laws of teleportation, teleportation with better performances can be implemented by selecting the appropriate time.

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I. INTRODUCTION

Quantum entanglement is the most counterintuitive physical phenomenon and plays very important roles because of its fundamental implications in quantum mechanics and its practical applications in quantum information and quantum communication [1]. Quantum teleportation [2] is the disembodied transmission of an unknown quantum state from the sender to the receiver using both entanglement and classical communication. Since Bennett *et al.*, proposed teleportation protocol which transports an unknown state of any discrete variable (DV) quantum system, many theoretical and experimental investigations of DV quantum teleportation were carried out [3–5]. Later, the original DV quantum teleportation was generalized to continuous variable (CV) domain [6–8] using Einstein-Podolsky-Rosen (EPR) entangled states [9]. The CV quantum teleportation, quantum teleportation of optical coherent states, was first demonstrated experimentally by Furusawa *et al.*, [10] using squeezed state entanglement. Experimental demonstration of teleportation of a squeezed thermal state was reported [11]. All these teleported quantum states are Gaussian states of single electromagnetic fields without considering the effects of external environments on teleportation performance.

In practice, the real-world success of CV teleportation depends on the longevity of entanglement states in two-mode or multimode quantum systems. The survival time of entanglement should be longer than the time needed for information processing. Unfortunately, the presence of decoherence in communication channels, which results from unavoidable interaction between systems and exterior environments, will degenerate entanglement. An analytic dynamical expression of entanglement between two noninteracting oscillators in both independent and common environments was derived by Vasile *et al.*, [12,13] using the characteristic function approach. It opens the way to quantitatively investigate effects of external environments on the performance of quantum communications using entanglement.

Most theoretical and experimental investigations of teleportation assume that the quantum channel is perfect and do not take the effects of external environment on teleportation performance into consideration. This is very unreasonable because whether the practical implementations of these quantum protocols are successful or not is unavoidably affected and even determined by the interactions between systems of interest and environments. In this paper, for the independent and common bosonic structured environment, we adopt the physical models provided by Vasile *et al.*, in [12] where only the weak system-environment coupling is assumed during the derivation process, and the Markovian approximation and rotating wave approximation are not assumed. We investigate the effects of environment and derive the analytical expression of fidelity which qualifies the transmission quality of CV teleportation in terms of both environmental and systematic parameters. Most of these effects are investigated in detail.

This paper is organized as follows. In Sec. II, the fidelity of quantum teleportation is derived in the terms of the elements of the evolved covariance matrix of CV two-mode squeezed states generated by nondegenerate optical parametric amplifiers. Thus the effects of environment on teleportation can be discussed by investigating the evolvement of a covariance matrix of two-mode Gaussian states. In Sec. III, we adopt the physical models proposed by Vasile *et al.*, [12,13], and according to the analytic expressions of fidelity obtained using the conclusions in Sec. II, the numerical simulations are given to show teleportation performance dependence on the environmental and systematic parameters. The conclusions are drawn in Sec. IV.

II. FIDELITY OF QUANTUM TELEPORTATION UNDER BOSONIC STRUCTURED ENVIRONMENTS

The performance of quantum communications which utilize the characteristics of quantum entanglement will inevitably be affected if quantum entanglement evolves under the exterior environments, one of which is bosonic structured environments, considered in this paper. As is well known, the effect of bosonic reservoirs on Gaussian states is equivalent to Gaussian

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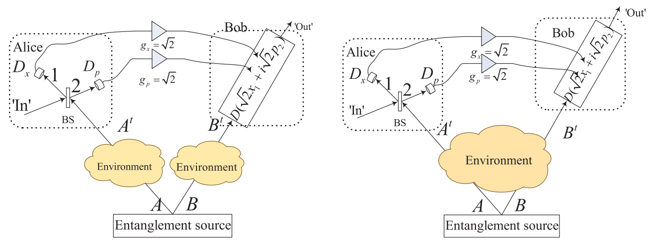

FIG. 1. (Color online) Quantum teleportation under effects of (a) independent bosonic environment and (b) common bosonic environment. Here BS denotes beam splitter ($\eta = 0.5$), and the letters and arabic numbers denote the modes. D_x and D_p are balanced homodyne detectors. $D(\alpha)$ is displacement operator. g_x and g_p are electrical amplifiers.

operations, which map one Gaussian state to another Gaussian state, while a Gaussian state could be well described by its characteristic function. So in order to study the performance of teleportation under bosonic structured environments, we first investigate teleportation using the characteristic function approach and give the fidelity expression of teleportation of a single-mode Gaussian state in terms of evolved covariance matrix elements of characteristic function.

The executive processes of CV teleportation under independent and common bosonic environments are depicted in Figs. 1(a) and 1(b), respectively. Alice and Bob first share an EPR type entangled state $\hat{\rho}_{\mathrm{EPR}}$ described by the following characteristic function:

$$\begin{aligned}\chi_{AB}(\lambda_A,\lambda_B) &:= \mathrm{Tr}[\rho_{AB} D(\lambda_A) D(\lambda_B)]\\ &= \exp\left\{-\tfrac{1}{2}\Lambda^T \sigma_{AB}\Lambda - i\Lambda^T \overline{X}_{AB}\right\}, \quad (1)\end{aligned}$$

where $D(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ is the displacement operator and $\hat{a}$ denotes the destroy operator, $\lambda_A = -\frac{i}{\sqrt{2}}(x_A + ip_A)$, $\lambda_B = -\frac{i}{\sqrt{2}}(x_B + ip_B)$, and $\Lambda = (x_A, p_A, x_B, p_B)^T$. Here we define the momentum operator and position one as $X_j = \frac{1}{\sqrt{2}}(\hat{a}_j + \hat{a}_j^\dagger)$, $P_j = -\frac{i}{\sqrt{2}}(\hat{a}_j - \hat{a}_j^\dagger)$, respectively. The covariance matrix σ_{AB} of mode A and mode B reads as

$$\sigma_{AB} = \begin{pmatrix} A_0 & C_0 \\ D_0 & B_0 \end{pmatrix}, \quad (2)$$

where $A_0 = aI$, $B_0 = bI$, $C_0 = \mathrm{diag}(c_1, c_2)$, $D_0 = \mathrm{diag}(d_1, d_2)$, with $a = b = \frac{1}{2}\cosh(2r)$, $c_1 = d_1 = \frac{1}{2}\sinh(2r)$, $c_2 = d_2 = -\frac{1}{2}\sinh(2r)$, and I is a 2×2 identity matrix. The average values read as

$$\overline{X}_{AB} = \mathrm{Tr}[\rho_{AB}(X_A, P_A, X_B, P_B)^T] = (0,0,0,0)^T. \quad (3)$$

The entangled optical modes A and B will suffer the interaction between system and environment before they are used for quantum teleportation. In particular, in some cases where the teleportation is not immediately implemented, quantum entanglements should be stored in quantum memories before they are used. The quantum state ρ_{AB} at time $t = 0$ will evolve to quantum state ρ_{AB}^t at time t according to the open-system master equation [14]. Because of the one-to-one corresponding relationship between quantum state ρ_{ij} and its characteristic function $\chi_{ij}(\lambda_i, \lambda_j)$ reading as

$$\rho_{ij} = \frac{1}{\pi^2}\int d^2\lambda_i\, d^2\lambda_j\, \chi_{ij}(\lambda_i,\lambda_j) D_i(-\lambda_i) D_j(-\lambda_j), \quad (4)$$

it is convenient to investigate the evolvement of entangled Gaussian states by studying that of the corresponding characteristic function. For Gaussian evolution such as the interaction between oscillators and bosonic environments which is used in this paper, the corresponding characteristic function $\chi_{AB}^t(\lambda_A^t, \lambda_B^t)$ still remains in its Gaussian form,

$$\chi_{AB}^t(\lambda_A^t, \lambda_B^t) = \exp\left\{-\tfrac{1}{2}\Lambda^T \sigma_{AB}^t \Lambda - i\Lambda^T \overline{X}_{AB}^t\right\}, \quad (5)$$

where the covariance matrix has the most general form, which certainly includes the case for interaction between systems and bosonic structured reservoirs,

$$\sigma_{AB}^t = \begin{pmatrix} A_t & C_t \\ D_t & B_t \end{pmatrix}, \quad (6)$$

where $A_t = \left(\begin{smallmatrix} A_{11} & A_{12} \\ A_{12} & A_{22}\end{smallmatrix}\right)$, $B_t = \left(\begin{smallmatrix} B_{11} & B_{12} \\ B_{12} & B_{22}\end{smallmatrix}\right)$, and $C_t = (D_t)^T = \left(\begin{smallmatrix} C_{11} & C_{12} \\ C_{21} & C_{22}\end{smallmatrix}\right)$. In the next section, the exact covariance matrix will be given for both an independent and common bosonic reservoir.

Here the average values have the following form:

$$\overline{X}_{AB}^t = \mathrm{Tr}\left[\rho_{AB}^t\left(X_A^t, P_A^t, X_B^t, P_B^t\right)^T\right] = (a,b,c,d)^T. \quad (7)$$

All of the above parameters have different expressions for different interaction styles in the different environments. The general form is presented for the convenience of obtaining the general expression of fidelity of teleportation for bosonic environments. All of the above parameters will have different values for the independent bosonic environment [see Fig. 1(a)] and common bosonic environment [Fig. 1(b)]. A beam splitter with transmittance 0.5 is used to combine mode A^t and "input" mode, producing modes 1 and 2. If the balanced homodyne detector is utilized to measure the quadrature operators X_1 and P_2 and the measurement results x_1 and p_2 work as the arguments of displacement operator $D(\sqrt{2}x_1 + i\sqrt{2}p_2)$ on the mode B^t kept by Bob, then the characteristic function of the "out" mode has the following form according to the theory of Marian *et al.*, [15],

$$\begin{aligned}\chi_{\mathrm{out}}(\lambda_{\mathrm{out}}) &= \chi_{\mathrm{in}}(\lambda_{\mathrm{out}})\chi_{AB}^t(\lambda_{\mathrm{out}}^*, \lambda_{\mathrm{out}})\\ &= \chi_{\mathrm{in}}(x_{\mathrm{out}}, p_{\mathrm{out}})\chi_{AB}^t(-x_{\mathrm{out}}, p_{\mathrm{out}}, x_{\mathrm{out}}, p_{\mathrm{out}}), \quad (8)\end{aligned}$$

where $\chi_{\mathrm{in}}(\lambda)$ is the characteristic function of the input pure coherent state to be teleported, with the covariance matrix $\sigma_{\mathrm{in}} = \left(\begin{smallmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2}\end{smallmatrix}\right)$ and $\overline{X}_{\mathrm{in}} = (\overline{x_{\mathrm{in}}}, \overline{p_{\mathrm{in}}})^T$.

According to Eqs. (5)–(8), the expression of $\chi_{\mathrm{out}}(\lambda_{\mathrm{out}})$ could be obtained as follows:

$$\chi_{\mathrm{out}}(x_{\mathrm{out}}, p_{\mathrm{out}}) = \exp\left\{-\tfrac{1}{2}\Lambda^T\sigma_{\mathrm{out}}\Lambda - i\Lambda^T\overline{X}_{\mathrm{out}}\right\}, \quad (9)$$

where the covariance matrix is expressed as

$$\sigma_{\mathrm{out}} = \begin{pmatrix} \frac{1}{2} + A_{11} + B_{11} - 2C_{11} & C_{21} + B_{12} - A_{12} - C_{12} \\ C_{21} + B_{12} - A_{12} - C_{12} & \frac{1}{2} + A_{22} + B_{22} + 2C_{22}\end{pmatrix}, \quad (10)$$

and the average value is expressed as

$$\overline{X}_{\rm out} = (c - a + \overline{x_{\rm in}}, b + d + \overline{p_{\rm in}})^T. \tag{11}$$

Thus we could directly obtain the expression of fidelity between the quantum states $\rho_{\rm in}$ and $\rho_{\rm out}$ which can be used to qualify the teleportation according to Marian's method [16,17],

$$F(\rho_{\rm in}, \rho_{\rm out}) = \frac{1}{\sqrt{MN - O^2}}, \tag{12}$$

where $M = 1 + A_{11} + B_{11} - 2C_{11}$, $N = 1 + A_{22} + B_{22} + 2C_{22}$, and $O = C_{21} + B_{12} - A_{12} - C_{12}$.

If we could obtain the exact expression of covariance matrix σ^t_{AB} of characteristic function of entangled states in Eq. (6), then the dynamical evolution of fidelity could be obtained in terms of environmental parameters, thus the effects of environment on teleportation can be quantitatively analyzed. In the next section, the exact physical models for interaction between systems of interest and environments are introduced.

III. NUMERICAL SIMULATIONS

As is well known, for different interactions in different types of environments, the covariance matrix has different expressions. Here we assume that two-mode squeezed vacuum states $\rho(t = 0) = \rho_{AB}$ are in bosonic structured reservoirs in either an independent environment or a common environment. The dynamical evolution processes of the corresponding characteristic function and their covariance matrix of two-mode squeezed states are investigated in detail according to the open-system master equation [14,18].

The performance of quantum teleportation is affected by the environment parameters including $J(\omega)$, T, and α, and the system parameters including ω_0 and r. Here the coupling coefficient is set as $\alpha = 0.1$, $k_B T/\hbar\omega_c = 100$ (high temperature). For the convenience of discussion, we define one important parameter, $x = \frac{\omega_c}{\omega_0}$, i.e., the ratio of the cut-off frequency of the spectral density $J(\omega)$ of the environment and the oscillator frequency of the quantum system. When $x \ll 1$, the evolution process is non-Markovian evolution, otherwise it is Markovian evolution. Here we set the squeezed factor of two-mode squeezed vacuum states as $r = 0.5$. The exact expression of σ^t_{AB} is shown in [12] and [13] as a function of the parameters above, and then we can discuss the dynamical evolution of the fidelity of teleportation in terms of the above four parameters for the following cases. Obviously, the communication performance of teleportation is related to the degree of entanglement; so it is reasonable to investigate the evolution of entanglement.

Here we introduce the calculation method of logarithmic negativity which is used to qualify the entanglement degree. For the two-mode Gaussian state, logarithmic negativity can be calculated by the following formula:

$$E_N = \max\{0, -\ln \widetilde{\nu}_-\}, \tag{13}$$

where $\widetilde{\nu}_- = 0.5\sqrt{I_1 + I_2 - 2I_3 - \sqrt{(I_1 + I_2 - 2I_3)^2 - 4I_4}}$, and $I_1 = \det[A_t]$, $I_2 = \det[B_t] I_3 = \det[C_t]$, and $I_4 = \det[\sigma^t_{AB}]$, where k is the lowest symplectic eigenvalue of the partial transposed (PT) state. According to an analytical

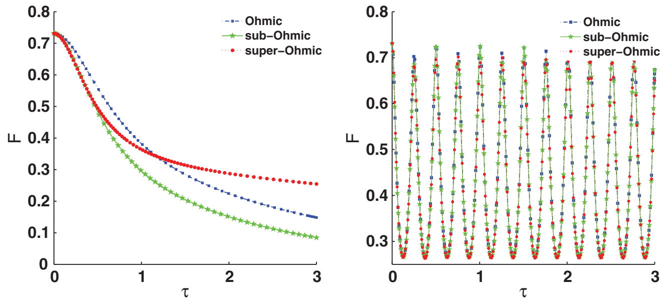

FIG. 2. (Color online) Fidelity of teleportation in an independent environment at high temperature: (a) for high $x = 10$; (b) for $x = 0.08$.

expression of fidelity and that of logarithmic negativity, we could discuss the effects of system parameters and environment parameters on both entanglement and the performance of quantum teleportation.

A. Independent environment

Here we analyze the effects of independent environment on the performance of quantum teleportation. In Fig. 2, we plot the fidelity as a function of $\tau = \omega_c t$ for Ohmic ($s = 1$), sub-Ohmic ($s = 0.5$), and super-Ohmic ($s = 3$) reservoirs for $x = 10$ [see Fig. 2(a)] and $x = 0.08$ [see Fig. 2(b)], respectively. In this case we noticed that the fidelity is a monotonically decreasing function of τ for $x = 10$, namely, $x \gg 1$, but it will oscillate for $x = 0.08$, namely, $x \ll 1$. It can also be found that three curves in Fig. 2(b) are nearly the same, meaning that when $x \ll 1$ the quality of teleportation is almost same in Ohmic, sub-Ohmic, and super-Ohmic.

B. Common environment

Next, we analyze the effects of common environment on the performance of quantum teleportation. In Fig. 3, we plot the fidelity for $x = 10$ [see Fig. 3(a)] and $x = 0.08$ [see Fig. 3(b)], respectively, both in a high temperature limit. The plots are similar to that of independent environment, namely, the fidelity is monotonically decreasing for $x \gg 1$ and oscillates for $x \ll 1$.

Comparing all the cases discussed above, we find that there are several interesting properties of fidelity of continuous-variable quantum teleportation in both independent and common environments. First of all, the ratio between the cutoff

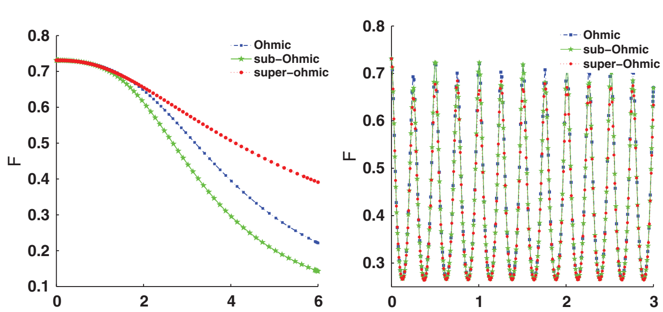

FIG. 3. (Color online) Fidelity of teleportation in common environment at high temperature: (a) for high $x = 10$; (b) for $x = 0.08$.

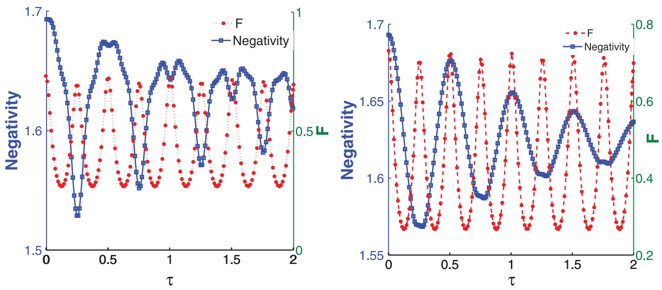

FIG. 4. (Color online) Relationship between fidelity of teleportation and negativity in the case of $x = 0.08$, $r = 0.5$: (a) for common environment and (b) for independent environment.

frequency of the environment spectrum ω_c and the oscillator frequency ω_0, namely, x in this paper, determines whether the fidelity is a monotone function of time or not. In the case of $x \gg 1$, i.e., for Markovian evolution, the fidelity is monotonically decreasing, while it oscillates when $x \ll 1$, i.e., for non-Markovian evolution. Second, when $x \ll 1$, the parameter s representing the environment spectrum does not affect the quality of teleportation very much.

C. Relationship between fidelity and degree of entanglement

Finally, we investigate the relationship between fidelity and the degree of entanglement. In general, a better entangled EPR state will implement quantum teleportation better. However, this is not always the case. Figure 4 illustrates such a situation. It is obvious that in both independent and common environments, at some time the fidelity is decreasing while the negativity is increasing if the same protocol is applied all the time. The reason for this phenomenon is that we do not calculate the optimal fidelity here. We assume that Alice and Bob will apply the protocol shown in Sec. II at any time. Although the protocol will achieve optimal fidelity at $t = 0$, it may not be the optimal one at other times.

IV. SUMMARY AND CONCLUSIONS

In this paper, we investigate the effects of dynamics of continuous variable entanglement on the performance of quantum teleportation. We have shown the fidelity as a function of time in a variety of cases. We find that generally, as time passes, the performance of quantum teleportation will become worse, and we analyze the contribution of the parameter x representing the ratio of cutoff frequency of the environment spectrum and the frequency of the oscillator, s representing the environment spectrum and whether the experiment is done in an independent reservoir or a common reservoir. It can be concluded that among all these parameters, x will affect the quality of teleportation most. Lastly, we compare the negativity and fidelity and show that if the same protocol is applied all the time, the fidelity may not change in the same way as negativity.

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