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Teleportation of continuous variable multimode Greeberger–Horne–Zeilinger entangled states

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Abstract

Quantum teleportation protocols of continuous variable (CV) Greeberger–Horne–Zeilinger (GHZ) and Einstein–Podolsky–Rosen (EPR) entangled states are proposed, and are generalized to teleportation of arbitrary multimode GHZ entangled states described by Van Loock and Braunstein (2000 *Phys. Rev. Lett.* **84** 3482). Each mode of a multimode entangled state is teleported using a CV EPR entangled pair and classical communication. The analytical expression of fidelity for the multimode Gaussian states which evaluates the teleportation quality is presented. The analytical results show that the fidelity is a function of both the squeezing parameter r, which characterizes the multimode entangled state to be teleported, and the channel parameter p, which characterizes the EPR pairs shared by Alice and Bob. The fidelity increases with increasing p, but decreases with increasing r, i.e., it is more difficult to teleport the more perfect multimode entangled states. The entanglement degree of the teleported multimode entangled states increases with increasing both r and p. In addition, the fact is proved that our teleportation protocol of EPR entangled states using parallel EPR pairs as quantum channels is the best case of the protocol using four-mode entangled states (Adhikari *et al* 2008 *Phys. Rev.* A **77** 012337).

1. Introduction

Quantum teleportation is the disembodied transmission of an unknown quantum state from the sender to receiver using both the quantum correlation called entanglement and classical communication [1]. It has been considered as one of the fundamental quantum operations in quantum computation and quantum information [2, 3]. Quantum computation with cluster states is a typical example [4]. Since Bennett *et al* proposed quantum teleportation which transports an unknown state of any discrete variable (DV) quantum system [1], many theoretical and experimental investigations of DV quantum teleportation were carried out [5–7]. Later, the original DV quantum teleportation was generalized to the continuous variable (CV) domain [8–10] using EPR entangled states [11]. The CV quantum teleportation, quantum teleportation of optical coherent states, was first demonstrated experimentally by Furusawa *et al* [12] using squeezed state entanglement. An experimental demonstration of teleportation of a squeezed thermal state was given in [13]. CV entanglement swapping protocols were implemented [14, 15]. The above CV schemes are teleportation protocols of Gaussian states. The CV quantum teleportation using non-Gaussian states of the radiation field as entangled resources was investigated by Dell'Anno *et al* [16]. The above schemes are about the teleportation of a single electromagnetic field; in fact the teleportation of a CV composite system, such as CV GHZ entangled states, is very important for quantum information processing [6]. As a similar scheme for teleportation of a two-qubit entangled state [7], the teleportation protocol of two-mode squeezed states using the four-mode entangled state was proposed by Adhikari *et al* [17]. But the experimental setup of Adhikari's scheme involving the four-mode entangled state is complex and is difficult to implement, and its analytical expression of fidelity has the maximal value 0.38 which is incomparable with the fidelity definition [18] with the maximal value 1. So designing simple teleportation protocol of CV EPR entangled states and presenting its proper fidelity expression

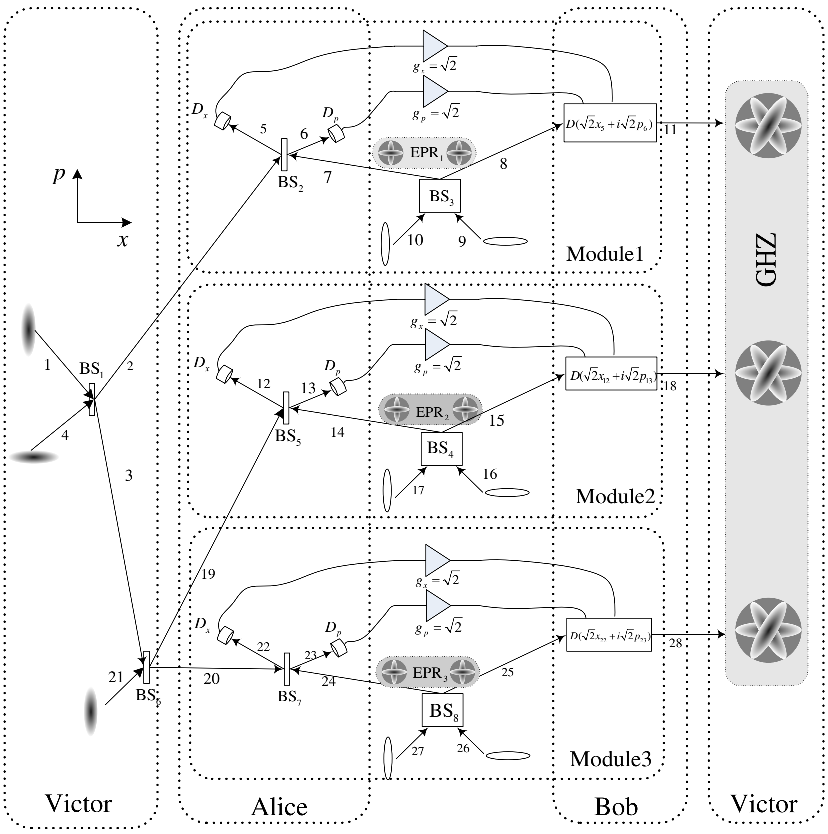

Figure 1. Schematic representation of quantum teleportation of CV GHZ entangled states. BS: beam splitter, *D*: displacement operator. The Arabic numbers denote the modes. The transmission parameter η of all beam splitters except BS_1 is 0.5.

are significant. In addition, to the best of our knowledge, the teleportation protocol of the CV GHZ entangled state has not been proposed in the literature.

The purpose of this paper is to present a teleportation protocol of an unknown CV GHZ entangled state from Alice to Bob, and to analyze the teleportation quality of our protocol by calculating the fidelity for multimode Gaussian states. Our protocol uses three pairs of CV EPR entangled states as quantum channel. In addition, the teleportation protocol of CV EPR entangled states is designed using two pairs of EPR entangled states, which is more easily prepared than the four-mode entangled state [17], and the explicit expression of our protocol's fidelity comparable with the fidelity definition [18] is given. Finally, we generalize the teleportation of the CV GHZ entangled states to that of arbitrary multimode GHZ entangled states generated by combining the squeezed states on beam splitters [19], and present the analytical expression of the fidelity in terms of both the squeezing parameter r and the channel parameter p.

This paper is organized as follows. In section 2, the teleportation protocol of CV GHZ entangled states is described. In section 3, the analytical expression of fidelity in terms of both r and p is given in order to evaluate the teleportation quality. In section 4, the teleportation protocol of CV EPR entangled states is proposed, and is compared with the scheme by Adhikari *et al* [17]. In section 5, the general teleportation of arbitrary multimode GHZ entangled states [19] is proposed and the analytical expression of fidelity is given. Finally, conclusions are drawn in section 6.

2. Teleportation protocol of CV three-mode GHZ entangled states

First, a third independent party, usually called Victor, prepares three pairs of CV EPR entangled states, and distributes those to the communication parties, Alice and Bob, as quantum channels. Then Alice teleports each of three modes in an unknown GHZ entangled state, which is supplied by Victor to Bob using a shared CV EPR pair and classical communication. The quantum teleportation protocol of CV GHZ entangled states may be described generally in the following steps (see figure 1).

Step 1: Victor prepares the CV GHZ entangled state [19] to be teleported. Victor first combines the 'position' quadrature $(X = \hat{a} + \hat{a}^{\dagger})$ squeezed mode $\hat{a}_1$ with the 'momentum' quadrature $\left(P = \frac{1}{i}(\hat{a} - \hat{a}^{\dagger})\right)$ squeezed mode $\hat{a}_4$ by beam splitter BS_1 with the transmission parameter $\eta = \frac{1}{3}$, producing the modes $\hat{a}_2$ and $\hat{a}_3$. Then Victor combines mode $\hat{a}_3$ with the 'position' quadrature squeezed mode $\hat{a}_{21}$ by beam splitter BS_6 to produce the modes $\hat{a}_{19}$ and $\hat{a}_{20}$. Here three modes $\hat{a}_2, \hat{a}_{19}$ and $\hat{a}_{20}$ are in the GHZ entangled state that Alice wants to teleport to Bob.

Step 2: Victor prepares three pairs of EPR entangled states as quantum channels for quantum teleportation of the GHZ entangled state. Victor prepares the EPR entangled modes,

$\hat{a}_7$ and $\hat{a}_8$, by combining the X squeezed mode $\hat{a}_{10}$ and the P squeezed mode $\hat{a}_9$ on the beam splitter BS_3. In a similar way, the entangled pairs $\hat{a}_{14}, \hat{a}_{15}$ and $\hat{a}_{24}, \hat{a}_{25}$ are prepared by Victor. Victor sends modes $\hat{a}_7, \hat{a}_{14}, \hat{a}_{24}$ to Alice, and sends modes $\hat{a}_8, \hat{a}_{15}, \hat{a}_{25}$ to Bob.

Step 3: Alice teleports modes $\hat{a}_2$, $\hat{a}_{19}$ and $\hat{a}_{20}$ using module 1, module 2 and module 3 respectively. Here we only describe the teleportation process of $\hat{a}_2$ (module 1). The teleportation of modes $\hat{a}_{19}$ and $\hat{a}_{20}$ can be described in a similar way. Alice combines $\hat{a}_2$ with one arm ($\hat{a}_7$) of EPR_1 pair by BS_2, obtaining modes $\hat{a}_5$ and $\hat{a}_6$. Alice first measures X_5 and P_6 by the homodyne measurement devices, then gives the measurement results to Bob by classical communication. Bob amplifies Alice's results with the gain parameters $g_x = g_p = \sqrt{2}$, and applies the displacement operator $D(\sqrt{2}X_5 + i\sqrt{2}P_6)$ on the other arm ($\hat{a}_8$) of EPR_1 pair, obtaining mode $\hat{a}_{11}$. Mode $\hat{a}_{11}$ is the teleported one of $\hat{a}_2$.

Step 4: Alice can teleport modes $\hat{a}_{19}$ and $\hat{a}_{20}$ to Bob by module 2 and module 3 respectively in a similar way, obtaining teleported ones $\hat{a}_{18}$ and $\hat{a}_{28}$. Obviously, the above protocol can be easily generalized to that of arbitrary multimode GHZ entangled states.

Step 5: the fidelity between the original GHZ entangled state and the teleported one is calculated in order to evaluate the transmission performance of the teleportation process.

3. Teleportation criterion and fidelity for multimode Gaussian states

In order to estimate the 'quality' of the teleportation protocol, the fidelity determining the overlap between the input state and the output state is adopted. In this section, we first give the calculation formula of fidelity for multimode Gaussian states, then calculate the covariance matrices of the input state and the output state respectively, finally giving the analytical expression of fidelity in terms of both the squeezing parameter r of the GHZ state and quantum channel parameter p.

3.1. Fidelity for multimode Gaussian states

Definition. *The quantum fidelity was first defined by Jozsa, based on Uhlmann's transition probability [20]. Given two quantum states ρ_1 and ρ_2, the fidelity is given [18, 21] by*

$$F(\rho_1, \rho_2) = \left[\mathrm{Tr}\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}\right]^2, \qquad (1)$$

where ρ_1 and ρ_2 represent the density matrices of the input state and the output state respectively, and F reaches its maximal value 1 *if and only if $\rho_1 = \rho_2$.*

Calculation formula. The general formula for the fidelity of multimode Gaussian states is as follows [18]:

$$F(\rho_1, \rho_2) = \sqrt{L \det \Phi(O)}, \qquad (2)$$

where

$$L = \left[\det \frac{A_1 + A_2}{2}\right]^{-1},$$

$$\Phi : A \to A(I + \sqrt{I + (JA)^{-2}}),$$

$$O = \Phi(A_1) - (\Phi(A_1) - iJ)[A_2 + \Phi(A_1) - (A_2 - iJ)(\Phi(A_1) + A_2)^{-1}(A_2 + iJ)]^{-1}(\Phi(A_1) + iJ),$$

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix},$$

here A_1 and A_2 are the covariance matrices of the input state and the output state respectively.

Although the fidelity can be theoretically calculated using equation (2), it is hard to find a concrete expression of the fidelity for multimode Gaussian states. Fortunately, it has been proved [18] that when A_1 represents a pure state we can get $\Phi(A_1) = A_1$ and $(\Phi(A_1) - iJ)[A_2 + \Phi(A_1) - (A_2 - iJ)(\Phi(A_1) + A_2)^{-1}(A_2 + iJ)]^{-1}(\Phi(A_1) + iJ) = 0$, so $O = A_1$. Thus calculation formula (2) of the fidelity reduces to

$$F = \frac{1}{\sqrt{\det \frac{A_1 + A_2}{2}}}. \qquad (3)$$

It is obvious that the CV GHZ entangled state is a pure state, so we can calculate the fidelity between input and output by applying equation (3). Obviously, we first calculate the covariance matrices A_1 and A_2 in order to calculate the fidelity.

3.2. The covariance matrices of the input state and the output state

For our teleportation protocol, the input modes are $\hat{a}_2, \hat{a}_{19}$ and $\hat{a}_{20}$, and the output modes are $\hat{a}_{11}, \hat{a}_{18}$ and $\hat{a}_{28}$. According to equation (3), we must first calculate the matrices A_1 and A_2 in order to obtain the fidelity. Here introducing the vector of operators as $A = (X_1, \ldots, X_n, P_1, \ldots, P_n)^{\mathrm{T}}$ for a system made of n bosons, the covariance matrix V is defined in the following way:

$$V_{kl} = [V]_{kl} = \tfrac{1}{2}\langle A_k A_l + A_l A_k\rangle - \langle A_k\rangle\langle A_l\rangle. \qquad (4)$$

The CV GHZ entangled state [19] can be expressed as follows in Heisenberg picture:

$$X_2 = \sqrt{\tfrac{1}{3}}\, e^{r} X_4^{(0)} + \sqrt{\tfrac{2}{3}}\, e^{-r} X_1^{(0)}, \qquad (5)$$

$$P_2 = \sqrt{\tfrac{1}{3}}\, e^{-r} P_4^{(0)} + \sqrt{\tfrac{2}{3}}\, e^{r} P_1^{(0)}, \qquad (6)$$

$$X_{19} = \sqrt{\tfrac{1}{2}}\, e^{-r} X_{21}^{(0)} - \sqrt{\tfrac{1}{6}}\, e^{-r} X_1^{(0)} + \sqrt{\tfrac{1}{3}}\, e^{r} X_4^{(0)}, \qquad (7)$$

$$P_{19} = \sqrt{\tfrac{1}{2}}\, e^{r} P_{21}^{(0)} - \sqrt{\tfrac{1}{6}}\, e^{r} P_1^{(0)} + \sqrt{\tfrac{1}{3}}\, e^{-r} P_4^{(0)}, \qquad (8)$$

$$X_{20} = -\sqrt{\tfrac{1}{6}}\, e^{-r} X_1^{(0)} + \sqrt{\tfrac{1}{3}}\, e^{r} X_4^{(0)} - \sqrt{\tfrac{1}{2}}\, e^{-r} X_{21}^{(0)}, \qquad (9)$$

$$P_{20} = -\sqrt{\tfrac{1}{6}}\, e^{r} P_1^{(0)} + \sqrt{\tfrac{1}{3}}\, e^{-r} P_4^{(0)} - \sqrt{\tfrac{1}{2}}\, e^{r} P_{21}^{(0)}, \qquad (10)$$

where $\langle \Delta X_i^{(0)}\rangle^2 = \langle \Delta P_i^{(0)}\rangle^2 = 1, i = 1, 4, 21$, i.e., $|\psi_1^{(0)}\rangle$, $|\psi_4^{(0)}\rangle$ and $|\psi_{21}^{(0)}\rangle$ being vacuum states, r being squeezing parameter of the GHZ state to be teleported. From equation (5) to (10), we can find that $X_2 = X_{19} = X_{20}$, $P_2 + P_{19} + P_{20} = 0$ when $r \to \infty$, thus the modes $\hat{a}_2, \hat{a}_{19}$ and $\hat{a}_{20}$ are in the CV GHZ entangled state.

According to equations (4)–(10), the covariance matrix of the input state A_1 can be calculated as follows:

$$A_1 = \begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix}, \tag{11}$$

where

$$A_{11} = \begin{pmatrix} \frac{1}{3}e^{2r} + \frac{2}{3}e^{-2r} & \frac{1}{3}e^{2r} - \frac{1}{3}e^{-2r} & \frac{1}{3}e^{2r} - \frac{1}{3}e^{-2r} \\ \frac{1}{3}e^{2r} - \frac{1}{3}e^{-2r} & \frac{1}{3}e^{2r} + \frac{2}{3}e^{-2r} & \frac{1}{3}e^{2r} - \frac{1}{3}e^{-2r} \\ \frac{1}{3}e^{2r} - \frac{1}{3}e^{-2r} & \frac{1}{3}e^{2r} - \frac{1}{3}e^{-2r} & \frac{1}{3}e^{2r} + \frac{2}{3}e^{-2r} \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} \frac{1}{3}e^{-2r} + \frac{2}{3}e^{2r} & \frac{1}{3}e^{-2r} - \frac{1}{3}e^{2r} & \frac{1}{3}e^{-2r} - \frac{1}{3}e^{2r} \\ \frac{1}{3}e^{-2r} - \frac{1}{3}e^{2r} & \frac{1}{3}e^{-2r} + \frac{2}{3}e^{2r} & \frac{1}{3}e^{-2r} - \frac{1}{3}e^{2r} \\ \frac{1}{3}e^{-2r} - \frac{1}{3}e^{2r} & \frac{1}{3}e^{-2r} - \frac{1}{3}e^{2r} & \frac{1}{3}e^{-2r} + \frac{2}{3}e^{2r} \end{pmatrix}.$$

By the standard calculation of teleportation of a single mode, the output modes can be written as

$$X_{11} = X_2 + \sqrt{2}\,e^{-p} X_{10}^{(0)}, \tag{12}$$

$$P_{11} = P_2 - \sqrt{2}\,e^{-p} P_9^{(0)}, \tag{13}$$

$$X_{18} = X_{19} + \sqrt{2}\,e^{-p} X_{17}^{(0)}, \tag{14}$$

$$P_{18} = P_{19} - \sqrt{2}\,e^{-p} P_{16}^{(0)}, \tag{15}$$

$$X_{28} = X_{20} + \sqrt{2}\,e^{-p} X_{27}^{(0)}, \tag{16}$$

$$P_{28} = P_{20} - \sqrt{2}\,e^{-p} P_{26}^{(0)}, \tag{17}$$

where p is the squeezing parameter of the parallel EPR pairs shared by Alice and Bob. Obviously, $X_{11} = X_{18} = X_{28}$, $P_{11} + P_{18} + P_{28} = 0$ when $r \to \infty$ and $p \to \infty$, which means that the entanglement degree of the teleported ones increases with increasing both r and p.

According to equations (12)–(17), the covariance matrix A_2 of output modes is obtained as

$$A_2 = A_1 + 2\,e^{-2p} I. \tag{18}$$

Thus the covariance matrices of the input state and the output state are obtained by equations (11) and (18) respectively.

3.3. Fidelity of teleportation for CV GHZ entangled states

To check the teleportation 'quality', we calculate fidelity according to equation (3). The analytical expression of fidelity is obtained by substituting equations (11) and (18) into equation (3),

$$F = \frac{1}{\sqrt{R}}, \tag{19}$$

where $R = (1 + e^{-2r-2p} + e^{2r-2p} + e^{-4p})^3$.

According to equation (19), here we numerically plot the fidelity of teleportation in terms of both r and p in figure 2. As we can see, the fidelity will generally increase with increasing p, which means that the better quantum channel (CV EPR entangled pair) provides the better teleportation quality. This means that the fidelity is exactly equivalent to the multipartite entanglement of N-mode symmetric states [22, 23], here N parallel EPR entangled pairs, serving as quantum channels. And the fidelity will decrease with

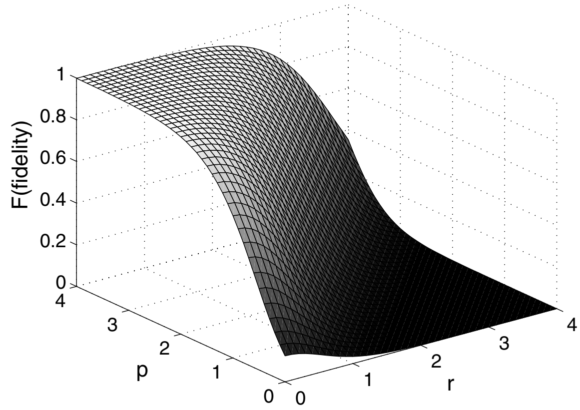

Figure 2. Teleportation fidelity of the GHZ entangled state in terms of p and r.

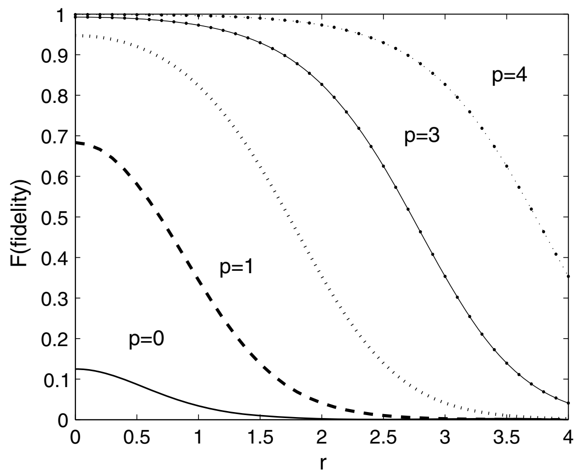

Figure 3. The relationship of fidelity with the squeezed parameter r, given $p = 0, 1, 2, 3, 4$.

increasing r, which indicates that it is more difficult to teleport the better CV GHZ entangled states, i.e. there is a trade-off between the fidelity and the entanglement degree of the teleported CV GHZ entangled state which is characterized by the squeezing parameter r. The above facts are also well illustrated in figures 3 and 4. Therefore the coherent state $r = 0$ is the best quantum signal once the quantum channel is built, i.e., p is a fixed value. The maximal value of fidelity is 1 when $r = 0, p \to \infty$, which means that the teleported mode is exactly the original one when Alice teleports the coherent state using the perfect quantum channel. According to equation (19), the fidelity of the classical 'teleportation' is $\frac{1}{8} = \left(\frac{1}{2}\right)^3$ with $r = 0$, $p = 0$, this case corresponds to the cascaded operation of three 'classical' teleportations of the coherent state with fidelity $F = 0.5$ [12, 24, 25].

In the above protocol, three parallel EPR entangled states serve as quantum channels. This is one of the quantum channels suitable for teleportation of the CV GHZ entangled state. One may find more quantum channels, for example the multi-party entangled state, for teleportation of multimode quantum states. But the following analysis shows that the other

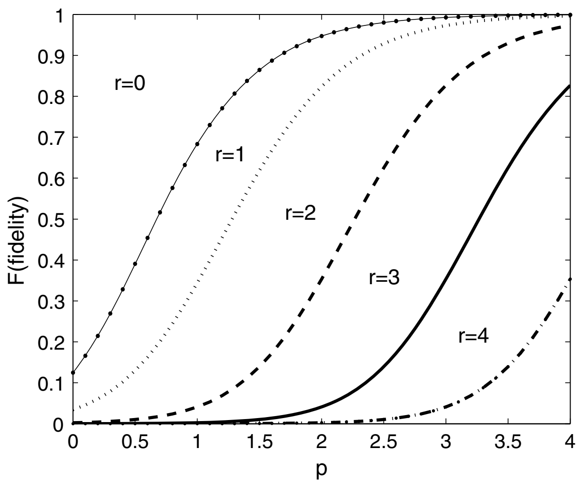

Figure 4. The relationship of fidelity with quantum channel parameter p, given $r = 0, 1, 2, 3, 4$.

quantum channels may be equivalent to the quantum channel using parallel EPR entangled states. If using parallel EPR entangled states as a quantum channel, the above teleportation protocol obviously can be easily generalized to teleportation for arbitrary multimode GHZ entangled states since one can teleport each mode by one EPR pair respectively.

4. Teleportation protocol of CV EPR entangled states

If we set up the transmission parameter of BS_1 as $\eta = \cos^2\theta$, omit module 3, and teleport mode $\hat{a}_3$ using module 2, then the above protocol reduces to that of CV EPR entangled states.

The EPR entangled pair can be expressed in Heisenberg representation as

$$X_2 = \cos\theta\,\mathrm{e}^{r} X_4^{(0)} + \sin\theta\,\mathrm{e}^{-r} X_1^{(0)}, \tag{20}$$

$$P_2 = \cos\theta\,\mathrm{e}^{-r} P_4^{(0)} + \sin\theta\,\mathrm{e}^{r} P_1^{(0)}, \tag{21}$$

$$X_3 = -\cos\theta\,\mathrm{e}^{-r} X_1^{(0)} + \sin\theta\,\mathrm{e}^{r} X_4^{(0)}, \tag{22}$$

$$P_3 = -\cos\theta\,\mathrm{e}^{r} P_1^{(0)} + \sin\theta\,\mathrm{e}^{-r} P_4^{(0)}. \tag{23}$$

Obviously, $\lim_{\substack{\theta=\frac{\pi}{4}\\ r\to\infty}} X_2 = X_3$, $\lim_{\substack{\theta=\frac{\pi}{4}\\ r\to\infty}} P_2 + P_3 = 0$, thus modes $\hat{a}_2$ and $\hat{a}_3$ are the entangled modes.

According to equations (20)–(23), the covariance matrix B_1 of input modes can be calculated as

$$B_1 = \begin{pmatrix} B_{11} & 0 \\ 0 & B_{22} \end{pmatrix}, \tag{24}$$

where

$$B_{11} = \begin{pmatrix} \cosh 2r + \cos 2\theta \sinh 2r & \sin 2\theta \sinh 2r \\ \sin 2\theta \sinh 2r & \cosh 2r - \cos 2\theta \sinh 2r \end{pmatrix},$$

$$B_{22} = \begin{pmatrix} \cosh 2r - \cos 2\theta \sinh 2r & -\sin 2\theta \sinh 2r \\ -\sin 2\theta \sinh 2r & \cosh 2r + \cos 2\theta \sinh 2r \end{pmatrix}.$$

The modes $\hat{a}_2$ and $\hat{a}_3$ are teleported to Bob using module 1 and module 2, respectively, by the standard teleportation process of a single electromagnetic field. The output modes

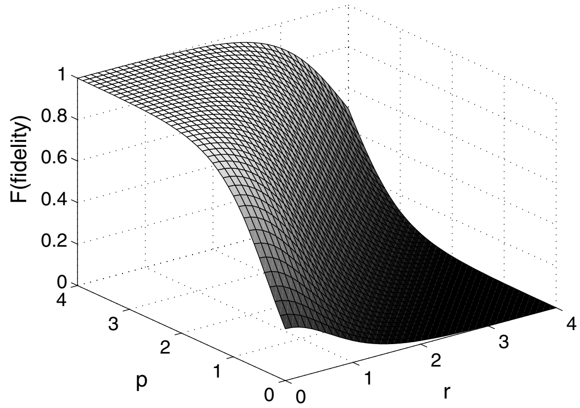

Figure 5. Teleportation fidelity of the EPR entangled state in terms of p and r.

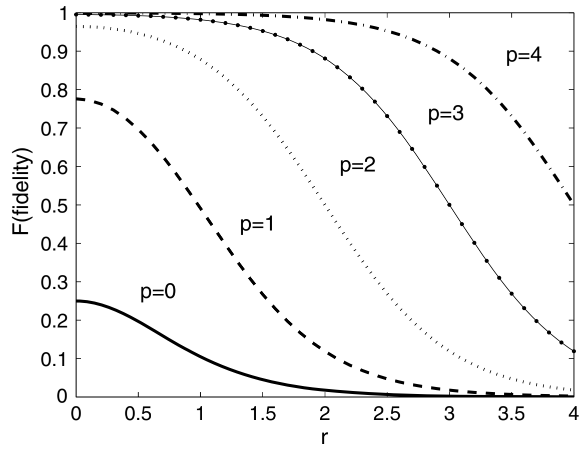

Figure 6. The relationship of fidelity with squeezed parameter r, given $p = 0, 1, 2, 3, 4$.

are expressed as

$$X_{11} = X_2 + \sqrt{2}\,\mathrm{e}^{-p} X_{10}^{(0)}, \tag{25}$$

$$P_{11} = P_2 - \sqrt{2}\,\mathrm{e}^{-p} P_9^{(0)}, \tag{26}$$

$$X_{18} = X_3 + \sqrt{2}\,\mathrm{e}^{-p} X_{17}^{(0)}, \tag{27}$$

$$P_{18} = P_3 - \sqrt{2}\,\mathrm{e}^{-p} P_{16}^{(0)}. \tag{28}$$

According to equations (25)–(28), the covariance matrix B_2 of output modes is calculated as

$$B_2 = B_1 + 2\,\mathrm{e}^{-2p} I. \tag{29}$$

By substituting equations (24) and (29) into equation (3), the fidelity is expressed as

$$F = \frac{1}{1 + \mathrm{e}^{-2r-2p} + \mathrm{e}^{2r-2p} + \mathrm{e}^{-4p}}. \tag{30}$$

From equation (30), we find that the fidelity F is a function of r and p, and is independent of θ, i.e., beam splitter does not affect the fidelity. F in terms of r and p is demonstrated in figures 5–7. As demonstrated in the figures, the fidelity also increases with increasing p which characterizes the multipartite entanglement, here being N pairs of EPR entangled states. So the fidelity is equivalent to multipartite

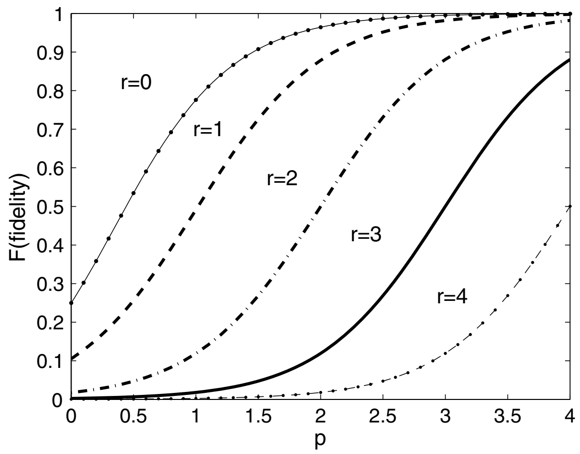

Figure 7. The relationship of fidelity with quantum channel parameter p, given $r = 0, 1, 2, 3, 4$.

entanglement, and this fact is compatible with the results of Adesso and Illuminati [22, 23]. And fidelity decreases with increasing r. So there also exists the trade-off between the teleportation quality and the entanglement degree of the CV EPR entangled states which is characterized by the squeezing parameter r.

In fact, Adhikari *et al* have investigated the teleportation protocol of the two-mode squeezed state using four-party entangled states as the quantum channel, and calculated the fidelity [17]. According to Adhikari's result, the relationship between the covariance matrix of the input mode and that of the output mode is as follows:

$$\sigma_{\text{out}} = \sigma_{\text{in}} + 2(c + ks)I, \tag{31}$$

where $c = \cosh 2p, k = \sin 2\phi, s = \sinh 2p$, p is the squeezing parameter of the quantum channel, ϕ is the squeezing phase. Obviously when $\phi = -\frac{\pi}{4}$,

$$\sigma_{\text{out}} = \sigma_{\text{in}} + 2\,\mathrm{e}^{-2p} I, \tag{32}$$

with the obtained fidelity reaching its maximal value. Formula (31) with the optimal value $\phi = -\frac{\pi}{4}$ is the same as equation (29) of our proposed protocol. That is to say, our protocol is the best case of Adhikari's protocol using four-mode entangled states with the same parameters r, p. Comparing with Adhikari's protocol using four-mode entangled states, our protocol uses relatively fewer optical devices, and is a better protocol more suitable for teleportation of two-mode squeezed states.

In addition, the maximal fidelity value of Adhikari's protocol is 0.38, which is incompatible with the fidelity definition [18, 21] with the maximal value 1. The reason is that Adhikari calculates the fidelity for the multimode Gaussian state using the fidelity formula for the single mode $F = \frac{1}{\sqrt{\det[\sigma_{\text{in}}+\sigma_{\text{out}}]+\delta}-\sqrt{\delta}}$, where $\delta = 4\left(\det[\sigma_{\text{in}}] - \frac{1}{4}\right)\left(\det[\sigma_{\text{out}}] - \frac{1}{4}\right)$, producing incompatible results.

5. Generalization to arbitrary multimode GHZ entangled states

In this section, we generalize the above protocols to teleportation of arbitrary multimode GHZ entangled Gaussian states [19]. In this paper, the vector of operators $A = (X_1, \ldots, X_n, P_1, \ldots, P_n)^T$ is adopted. By applying numbers of beam splitters $\hat{N}_{1\cdots N} = \hat{B}_{N-1N}\left(\frac{\pi}{4}\right)\hat{B}_{N-2N-1}\left(\cos^{-1}\frac{1}{\sqrt{3}}\right) \times \cdots \times \hat{B}_{12}\left(\cos^{-1}\frac{1}{\sqrt{N}}\right)$ on N-independent squeezed states with the covariance matrix $A_0 = \text{diag}(\mathrm{e}^{-2r_1}, \mathrm{e}^{2r_2}, \ldots, \mathrm{e}^{2r_1}, \mathrm{e}^{-2r_2}, \ldots)$, one can get the multimode GHZ entangled Gaussian states with the covariance matrix A_1 [19]. These N-mode GHZ entangled states can be teleported by N parallel EPR pairs and classical communication. The relationship between the covariance matrix of the input multimode entangled states A_1 and that of the output ones A_2 is as follows:

$$A_2 = A_1 + 2\,\mathrm{e}^{-2p} I, \tag{33}$$

where $A_1 = SA_0S^T$, $S \in Sp_{(2N,R)}$ is the symplectic operator corresponding to $\hat{N}_{1\cdots N}$ in phase space [26, 27].

To calculate the fidelity, we first prove two important properties of S: $SS^T = S^TS = I$ and $\det S = \det S^T = 1$. The derivation is as follows. Obviously S can be expressed as follows:

$$S = \prod_{k=2}^{N} B_k, \tag{34}$$

with

$$B_k = \begin{pmatrix} B_{k_x} & 0 \\ 0 & B_{k_p} \end{pmatrix},$$

$$B_{k_x} = B_{k_p} = \begin{pmatrix} I_{N-k} & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & I_{k-2} \end{pmatrix},$$

$$Q = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix},$$

where I_m is an m-order identity matrix.

Obviously, $QQ^T = Q^TQ = I$, $\det Q = \det Q^T = -1$, thus $B_kB_k^T = B_k^TB_k = I$, $\det B_k = \det B_k^T = 1$, then $SS^T = S^TS = I$, $\det S = \det S^T = 1$.

Using the properties of S, the fidelity of teleportation of arbitrary GHZ multimode entangled states is easily given as follows by substituting equation (33) into equation (3):

$$F = \frac{1}{\sqrt{M}}, \tag{35}$$

where $M = \det\frac{A_1+A_2}{2} = \det(A_1 + \mathrm{e}^{-2p}I) = \det[S(A_0 + \mathrm{e}^{-2p}I)S^T] = \det(A_0 + \mathrm{e}^{-2p}I) = \prod_{k=1}^{N}[(\mathrm{e}^{-2r_k} + \mathrm{e}^{-2p})(\mathrm{e}^{2r_k} + \mathrm{e}^{-2p})]$

When $r = r_k, k = 1, \ldots, N$, equation (35) reduces to

$$F = \frac{1}{\sqrt{[(\mathrm{e}^{-2r} + \mathrm{e}^{-2p})(\mathrm{e}^{2r} + \mathrm{e}^{-2p})]^N}}. \tag{36}$$

From equation (36), we can find that teleportation of N-mode GHZ entangled states generated by combining the squeezed states with beam splitters is equivalent with the cascaded teleportation of single mode squeezed states with the fidelity $F_{\text{sq}} = (\mathrm{e}^{-2r} + \mathrm{e}^{-2p})(\mathrm{e}^{2r} + \mathrm{e}^{-2p})^{-\frac{1}{2}}$. Obviously, the fidelity of GHZ states equation (19) and that of EPR states equation (30) are two special cases of equation (36) with $N = 3$ and $N = 2$ respectively.

According to equation (36), F still increases with increasing p for a certain r; this shows that the fidelity exactly corresponds to multipartite entanglement characterized by p, comparable with the results [22, 23]. While F decreases with increasing r, this indicates again that there exists a trade-off between the teleportation quality and the entanglement degree, which is characterized by r, of the multimode entangled states to be teleported. The above phenomena may be explained intuitively as follows. The quantum channels improve as the entanglement in the parallel EPR channels p grows, the quantum signal will be transmitted from Alice to Bob more precisely, and the resulting fidelity will increase. In another aspect, the improved entanglement of the input entangled states corresponds to the bigger squeezing parameter r; this means that the input entangled states contain more energy; it is obviously more difficult to transmit the more energy once the quantum channel is fixed. For example, when the quantum channel parameter p is a finite value, r is infinite for perfect entanglement, and it contains infinite energy, so it is difficult to perfectly transmit the infinite energy through such a channel corresponding to finite energy; thus the fidelity is very small.

6. Conclusion

In this paper, two kinds of quantum teleportation protocols of CV multimode entangled states, CV EPR and GHZ entangled states, are proposed, and are generalized to teleportation of arbitrary multimode GHZ entangled Gaussian states generated by combining the squeezed states on beam splitters. Each mode of the multimode entangled state is teleported using both an EPR entangled pair and classical communication. The analytical expression of fidelity which evaluates the teleportation quality of our protocol is presented. The analytical results show that the fidelity is a function of both the squeezing parameter r, which characterizes the multi-mode GHZ entangled state to be teleported, and the channel parameter p, which indicates the EPR pairs shared by Alice and Bob. The fidelity increases with increasing p, but decreases with increasing r, which indicates that there exists a trade-off between fidelity (transmission performance) and the entanglement degree, which is characterized by r, of the multimode GHZ entangled states to be teleported. We have proved that our protocol of EPR entangled states using parallel CV EPR pairs is the optimal case of Adhikari's scheme [17].

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References

[1] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[2] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[3] Braunstein S L and Pati A K 2003 *Quantum Information with Continuous Variables* (Dordrecht: Kluwer)
[4] Menicucci N C, van Loock P, Gu M, Weedbrook C, Ralph T C and Nielsen M A 2006 *Phys. Rev. Lett.* **97** 110501
[5] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575
[6] Zhang Q, Goebel A, Wagenknecht C, Chen Y A, Zhao B, Yang T, Mair A, Schmiedmayer J and Pan J W 2006 *Nature Phys.* **2** 678
[7] Yeo Y and Chua W K 2006 *Phys. Rev. Lett.* **96** 060502
[8] Vaidman L 1994 *Phys. Rev.* A **49** 1473
[9] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869
[10] Ralph T C and Lam P K 1998 *Phys. Rev. Lett.* **81** 5668
[11] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[12] Furusawa A, Sorensen J L, Braustein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[13] Takei N, Hiraoka T, Mizuno J, Takeoka M, Ban M and Furusawa A 2005 *Phys. Rev.* A **72** 042304
[14] van Loock P and Braunstein S L 1999 *Phys. Rev.* A **61** 010302(R)
[15] Jia X J, Su X L, Pan Q, Gao J R, Xie C D and Peng K C 2004 *Phys. Rev. Lett.* **93** 250503
[16] Dell'Anno F, Siena S D, Albano L and Illuminati F 2007 *Phys. Rev.* A **76** 022301
[17] Adhikari S, Majumdar A S and Nayak N 2008 *Phys. Rev.* A **77** 012337
[18] Paraoanu G S and Scutaru H 2000 *Phys. Rev.* A **61** 022306
[19] van Loock P and Braunstein S L 2000 *Phys. Rev. Lett.* **84** 3482
[20] Uhlmann A 1976 *Rep. Math. Phys.* **9** 273
[21] Scutaru H 1998 *J. Phys. A: Math. Gen.* **31** 3659
[22] Adesso G and Illuminati F 2005 *Phys. Rev. Lett.* **95** 150503
[23] Adesso G and Illuminati F 2007 *Phys. Rev. Lett.* **99** 150501
[24] Hammerer K, Wolf M M, Polzik E S and Cirac J I 2005 *Phys. Rev. Lett.* **94** 150503
[25] Adesso G and Chiribella G 2008 *Phys. Rev. Lett.* **100** 170503
[26] Laurat J, Keller G, Oliveira-Huguenin J A, Fabre C, Coudreau T, Serafini A, Adesso G and Illuminati F 2005 *J. Opt. B: Quantum Semiclass. Opt.* **7** S577
[27] Adesso G and Illuminati F 2007 *J. Phys. A: Math. Theor.* **40** 7821