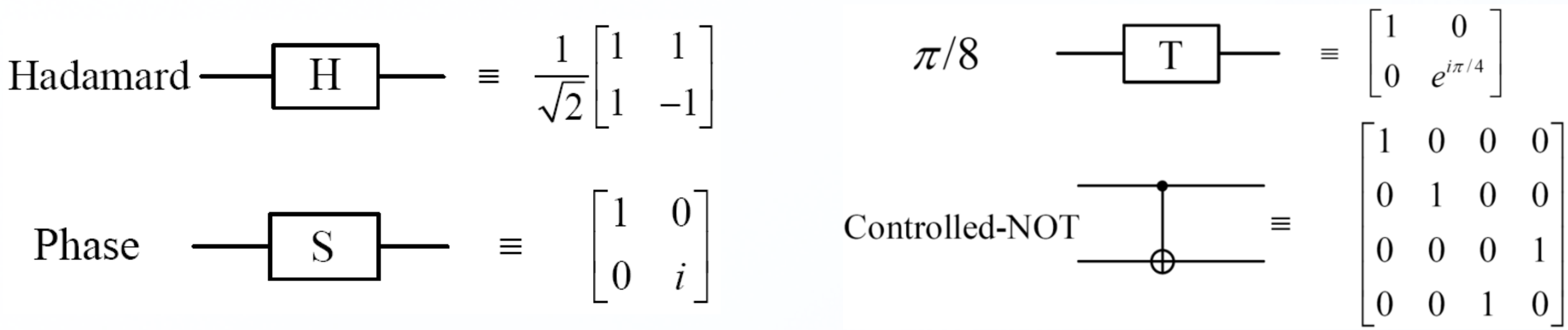
Universal quantum computation over discrete variable

- Universality: Any unitary operation on n qubits may be implemented exactly by composing single qubit and controlled-NOT gates.
- Universality with a discrete set: The Hadamard gate, phase gate, controlled-NOT gate, and $\pi/8$ gate are universal for quantum computation, in the sense that an arbitrary unitary operation on n qubits can be approximated to an arbitrary accuracy using a circuit composed of only these gates.



$$\text{Hadamard} \equiv \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \pi/8 \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$$\text{Phase} \equiv \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \text{Controlled-NOT} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information Cambridge University Press (2000).

Universal quantum computation over continuous variable

- Universality: Universal quantum computer over continuous variable can be constructed by a circuit composed of both Clifford operations $\{X(s), P(\eta), F, C_z\}$ and any non-Gaussian operation, e.g., photon count measurement or Kerr operation.

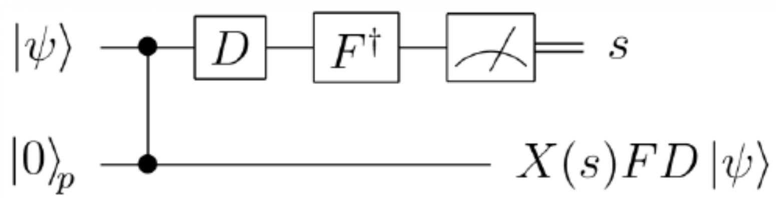
$$X(s) = \exp(-is\hat{p}) \qquad F = \exp[i\frac{\pi}{4}(\hat{x}^2 + \hat{p}^2)]$$

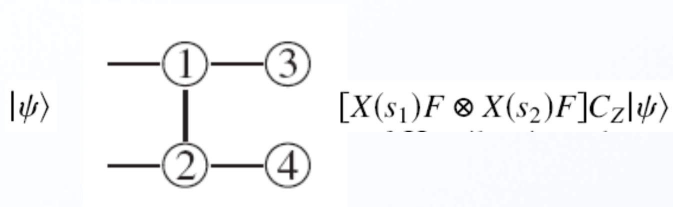
$$P(\eta) = \exp(\frac{i}{2}\eta\hat{x}^2) \qquad C_z = \exp(i\hat{x}_1\hat{x}_2)$$

$$\hat{x} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger), \hat{p} = \frac{1}{\sqrt{2}i}(\hat{a} - \hat{a}^\dagger)$$

1. S. Lloyd and S. L. Braunstein, Phys. Rev. Lett., 82, 1784(1999).
2. S. D. Bartlett et al, Phys. Rev. Lett., 88, 097904 (2002).

- Recently, universal quantum computation with continuous variable graph sates, which is a special stabilizer state, is proposed.





N. C. Menicucci et al, Phys. Rev. Lett., 97, 110501(2006).