Generation of Path-Frequency Hyperentanglement by Simultaneous Multiple Quasi-Phase Matching in Nonlinear Photonic Crystals

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Abstract: By using multiple quasi-phase matching in the nonlinear photonic crystals (NPC) whose poling direction is perpendicular to the pump beam, we theoretically generated path-frequency hyperentangled photon pairs.

1. The State Generated in the NPC

We will follow a standard approach in deriving the generated state [1]. We assume a strong classical incident pump under the lossless approximation, and spontaneous parametric down-conversion (SPDC) process happens in the NPC. The interactions are so weak that we can only care about the 2-photons states generated. In the interaction picture, the generated states can be expressed as:

$$|\psi(t\to+\infty)\rangle \approx |\psi(t\to-\infty)\rangle - \frac{i}{\hbar}\int_{-\infty}^{+\infty}\hat{H}_{int}(t)\,|\psi(t\to-\infty)\rangle dt\,, \qquad (1)$$

with a vacuum initial state. The nonlinear interaction Hamiltonian can be expressed using the classical pump field and the field operators for the signal and idler beams:

$$\hat{H}_{\text{int}}(t) \propto \varepsilon_0 \int d^3\vec{r}\,\chi^{(2)}(\vec{r})E_P^+(\vec{r},t)\hat{E}_S^-(\vec{r},t)\hat{E}_I^-(\vec{r},t) + H.c. \qquad (2)$$

After simplification and some approximation, we obtain the generated state:

$$|\psi\rangle \propto \sum_{\vec{k}_S}\sum_{\vec{k}_I} A_P\sqrt{\omega_S\omega_I}\,\frac{e^{(i\Delta k_z-\alpha_P)L}-1}{i\Delta k_z-\alpha_P}\int d^3\vec{r}\,\chi^{(2)}(x)e^{\Delta k_x x}\vec{e}_P\vec{e}_S^{\,*}\vec{e}_I^{\,*}\ \hat{a}_{k_S}^{\dagger}\hat{a}_{k_I}^{\dagger}\,|0\rangle_S\,|0\rangle_I\,, \qquad (3)$$

where $\hat{a}_{k_m}^{\dagger}$ (m=S, I) is the creation operator for signal or idler photon with the corresponding wave vector.

2. Our Entangled Photon Source Scheme Based on NPC

NPC has been studied to generate entangled photons or some specific quantum states, and theoretical analysis of NPC with higher dimensions like 2D NPC or 3D NPC has also been studied [2,3]. However, technical difficulties are limiting real experiments and applications. Our scheme uses 5% MgO-doped $LiNbO_3$ crystal in room temperature (25°C) to generate entangled photons. Specifically, we use the pattern to generate path-frequency hyperentanglement. The $\chi^{(2)}$ only depends on the x coordinate (shown in Fig. 1) so it is easier to launch experiments. From the expression of the generated state we know that as long as the crystal's $\chi^{(2)}(x)$ compensates the momentum mismatch in the x direction, the nonlinear process will happen, and the mismatch in the z direction has an impact on the conversion efficiency. Our expected parametric down-conversion (PDC) is type-I which means the pump beam is extraordinary light while the signal and idler beam are ordinary light.

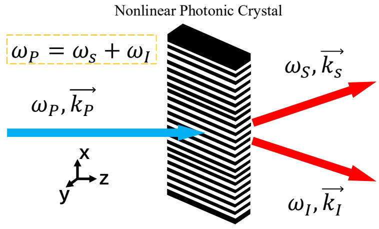

Fig. 1. Our scheme of entanglement source.

3. Two Designs of NPC to Generate Entanglement

Here we provide two different designs of NPC. The former will lead to path-frequency hyperentanglement while the latter will lead to path-frequency entanglement in a single photon. The pump wavelength is set at λ_P=775nm.

3.1. *Two-photon path-frequency hyperentanglement*

A hyperentanglement can be expressed as the tensor product of two entangled states:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\theta_{1,U},\theta_{1,D}\rangle + |\theta_{2,U},\theta_{2,D}\rangle) \otimes \frac{1}{\sqrt{2}}(|\omega_{1,U},\omega_{2,D}\rangle + |\omega_{2,U},\omega_{1,D}\rangle). \quad (4)$$

Here, U means up (above the pump) and D means down (below the pump). Fig. 2. (a) shows the momentum mismatch. In this case, four processes have momentum mismatches in the direction of z. Consequently, the PDC efficiency will be lower than the other cases. We ensure that the mismatches along the z-axis have the same numerical value but opposite direction, so the theoretical efficiency will be the same which means a maximum entanglement. Here we choose λ_S=1700nm, $\lambda_I \approx$1424.3nm. The corresponding Δk_{x1}=0.2801μm^{-1}, Δk_{x2}=0.3185μm^{-1}. Fig. 3. (a) shows our design.

3.2. *Single Photon entanglement in the dimension of path and frequency*

We can obtain a single photon whose path and frequency are entangled. Fig. 2. (b) shows the momentum mismatch of the PDC process. Once a photon in the direction of pump whose frequency is different from ω_P is detected, we know that the path-frequency entangled photon appears, just like a heralded single-photon source. Mathematically, the single photon can be expressed as:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\beta_1\rangle|\omega_1\rangle + |\beta_2\rangle|\omega_2\rangle), \quad (5)$$

where β_m (m=1,2) refers to the exit angles, ω_m(m=1,2) $\in \{\omega_S, \omega_I\}$. Here we choose λ_S=1700nm, $\lambda_I \approx$1424.3nm. The corresponding Δk_{x1}=2.228μm^{-1}, Δk_{x2}=2.443μm^{-1}. Fig. 3. (b) shows our design.

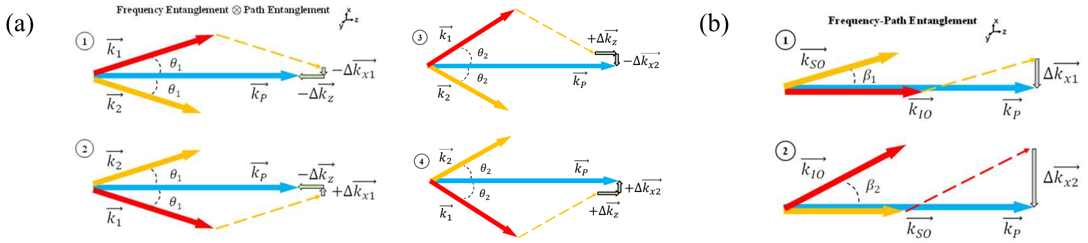

Fig. 2. Momentum mismatch of two types of entanglement.

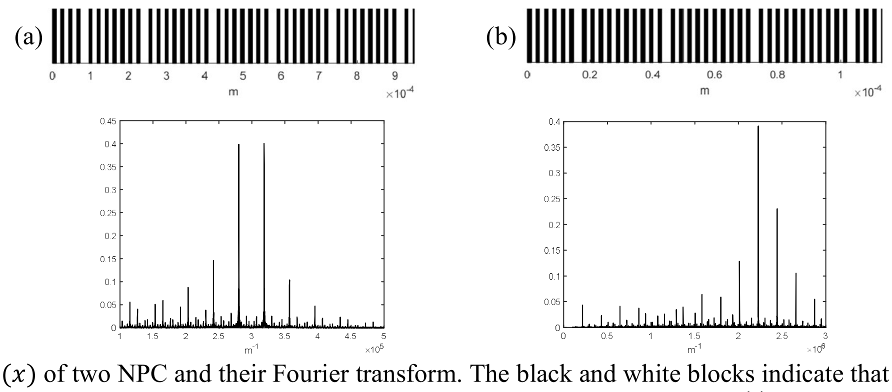

Fig. 3. $\chi^{(2)}(x)$ of two NPC and their Fourier transform. The black and white blocks indicate that $\chi^{(2)} = +1$ and $\chi^{(2)} = -1$, respectively. The PDC efficiency is proportional to the amplitude of $\chi^{(2)}(x)$'s Fourier transform at specific point.

4. References

[1] Saravi, Sina, Thomas Pertsch, and Frank Setzpfandt. "Generation of counterpropagating path-entangled photon pairs in a single periodic waveguide." Physical review letters 118.18 (2017): 183603.

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