### **Topologically Protected Entangled Photon Pairs in Honeycomb Photonic Crystals**

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**Abstract:** We propose a topological insulator supporting topologically protected entangled photon pairs. The continuous frequency entanglement between photon pairs generated from the four-wave mixing process is theoretically clarified. © 2022 The Author(s)

### 1. Introduction

Topological insulators have become a promising platform for implementing topologically protected classical or nonclassical states. The concepts of topological phases bring prominent properties such as robustness against imperfections to quantum systems. Very recently, advanced approaches have enriched topological quantum optics, including the topological single quantum emitters [1], topological quantum sources [2,3] and topological quantum entanglement [4].

Here we conduct topologically protected entangled photon pairs in topological photonic crystals. By emulating the quantum spin Hall effect (QSH) effect, topological edge states show robustness against the sharp bends and defects. Besides, the quantum entanglement between signal and idler photons generated from the FWM process is clarified.

### 2. Topological edge states

Topological edge states are characterized by the insulating bulk and conducting edge. As shown in Fig. 1(a), the topological photonic crystals (PCs) exciting the QSH effect are composed of deformed honeycomb lattices, where *R* is the radius of hexagon cluster, *r* is the radius of each silicon rod, *a* is the lattice constant. When an unperturbed honeycomb lattice is deformed to R = 0.9 a/3 (R = 1.1 a/3) without breaking the  $C_6$  crystal symmetry, the band structure shows trivial (nontrivial) topological properties. The dispersion relation of PCs composed of trivial and nontrivial lattices is depicted in Fig. 1(b). A pair of edge states occur within the topological bandgap, which refers to pseudospin-up ( $\sigma^+$ ) states and pseudospin-down ( $\sigma^-$ ) states respectively. Notably, different pseudospins correspond to counter-propagating direction. To get insight into the topological nature of edge states, we numerically simulate the electric field profiles of the FWM process in proposed PCs. As shown in Fig. 1(c), the electric fields of topological edge states are confined around the topological interface.



Fig. 1. (a) A scheme of topological PCs. (b)Dispersion relation of proposed PCs. (c) Field profiles of the topological edge states.

### 3. Topologically protected entangled photon pairs

Due to the intrinsic third-order nonlinear susceptibility tensor  $\chi^{(3)}$  of silicon, the FWM process is implemented with the excitation of the pump. It is noted that the degenerate FWM process should satisfy the relations:  $2\hbar\omega_p = \hbar\omega_s + \omega_s$ 

 $\hbar\omega_i$  and  $2k_p = k_s + k_i$ , where  $k_p$ ,  $k_s$  and  $k_i$  are wavevectors of the pump, signal, and idler respectively. Specifically, the signal and idler photons generated via the nonlinear FWM process may lead to entangled states. We note that the dispersion of topological edge states is linear, the wavevector mismatch  $\Delta k = 2k_p - k_s - k_i$  can be limited to a small value, which provides a practical way to conduct the broadband FWM process. Since the frequencies of the pump, signal and idler are localized inside the topological bandgap, the FWM process is topologically protected.

To theoretically manipulate the continuous frequency entanglement of photon pairs, we calculate the joint spectral amplitude (JSA) of biphoton states. The entanglement of the biphoton state  $|\Psi\rangle$  is clarified when JSA  $\mathcal{A}(\omega_s, \omega_i)$  cannot be factorized into two parts of  $\omega_s$  and  $\omega_i$ . The biphoton state is given by

$$\Psi = \iint d\omega_{s} d\omega_{i} \mathcal{A}(\omega_{s}, \omega_{i}) \hat{a}^{\dagger}_{\omega_{s}} \hat{a}^{\dagger}_{\omega_{i}} |0\rangle, \qquad (1)$$

where  $\mathcal{A}(\omega_s, \omega_i)$  is the JSA,  $\hat{a}^{\dagger}_{\omega_s}$  is the creation operator. Taking into consideration of the phase-matching condition of the nonlinear FWM process, the JSA can be written as

$$\mathcal{A}(\omega_s,\omega_i) = \alpha(\frac{\omega_s + \omega_i}{2}) \Phi(\omega_s,\omega_i), \tag{2}$$

with the spectrum envelope of the pump is  $\alpha(\frac{\omega_s + \omega_i}{2}) = \delta(\omega_s + \omega_i - 2\omega_p)$ , and the joint phase-matching spectrum is given by  $\Phi(\omega_s, \omega_i) = \operatorname{sinc}(\frac{\Delta kL}{2})$ .

The JSA describing the spectral correlations between photon pairs is depicted in Fig. 2(a), which reveals that the joint spectral intensity is strongly anticorrelated. We employ Schmidt decomposition to testify the separability of the JSA, the result of the normalized Schmit coefficient  $\lambda_n$  is plotted in Fig. 2(b). The number of nonzero Schmit coefficient  $\lambda_n$  is greater than 1, giving clear evidence of continuous frequency entanglement of photon pairs generated from the FWM process in topological PCs. Due to the topological nature of the QSH effect, the entangled photon pairs propagating along the interface are topologically protected, showing robustness against sharp bends and structure imperfections.



Fig. 2. (a) JSA of photon pairs generated from the FWM process in topological PCs. (b) Normalized Schmidt coefficients of photon pairs.

### 4. Conclusion

Here, we demonstrate a scheme of topological PCs implementing continuous frequency entangled photon pairs. We theoretically calculate the JSA and normalized Schmit coefficient of photon pairs generated via the FWM process in topological PCs. Our proposal may pave the way for unprecedented topological quantum devices.

The authors acknowledge the support from the Key-Area Research and Development Program of Guangdong Province (Grant 2018B030325002), and the National Natural Science Foundation of China (Grants 62075129/61975119).

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**JW3B.82** 

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Fig.2 (a) JSA and (b) normalized Schmidt coefficients.

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