Continuous variable multipartite entanglement and its applications & new measurement method

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Introduction

- Continuous variable (CV) multipartite entanglement plays important roles in quantum computation and quantum communication network.
- It is of great significance to investigate the characteristics of stabilizer state, one of which is graph state, under local Clifford (LC) operations.
- Entanglement degree of stabilizer state under LC operations remains unchanged. One purpose of our work is to determine whether two stabilizer state are equivalent or not under LC operations.
- Another purpose is to investigate quantum network using CV graph state.
- Finally, a new measurement method (R parameter) of entanglement degree of two partite CV entangled state is calculated using Wigner function, and R parameter is proved to be same as both Schmidt number K and F parameter related with Reid's method.

Equivalence of CV stabilizer states under local Clifford operations

- Stabilizer state: An n-mode stabilizer state $|\Psi\rangle$ is defined as the simultaneous eigenstate with eigenvalue 1 of n commutable and independent Pauli group elements.The set $S = \{G \in \mathcal{C}_1, G|\Psi\rangle = |\Psi\rangle\}$ is called the stabilizer of the state $|\Psi\rangle$.
- Local Clifford operations:

$$X(s) = \exp[-is\hat{p}] \quad (1)$$
$$Z(t) = \exp[it\hat{x}] \quad (2)$$
$$F = \exp[i(\pi/4)(\hat{x}^2 + \hat{p}^2)] \quad (3)$$
$$P(\eta) = \exp[i(\eta/2)\hat{x}^2] \quad (4)$$

- An element G of $\mathcal{C}_1$ can be generally written as the following form,

$$G = e^{i\theta}\prod_k X_k(s_k)Z_k(t_k), G \in \mathcal{C}_1, \theta \in [0, 2\pi), \quad (5)$$

- The mapping σ between the n-mode Pauli group and the set of $2n$-dimension real column vectors is defined as

$$\sigma(G) = \sigma[e^{i\theta}\prod_k X_k(s_k)Z_k(t_k)] \triangleq (t_1, ..., t_n, s_1, ..., s_n)^T. \quad (6)$$

- Corresponding to mapping σ, mapping δ maps the n-mode local Clifford $\mathcal{C}_2^n$ group to the set M^n, consisting of some $2n \times 2n$ real matrices. First consider one-mode local Clifford group $\mathcal{C}_2^1$ and the set M^1.

$$\delta(F) \triangleq \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \delta(P(\eta)) \triangleq \begin{bmatrix} 1 & \eta \\ 0 & 1 \end{bmatrix}, \quad (7)$$
$$\delta(X(u)) \triangleq I, \delta(Z(v)) \triangleq I. \quad (8)$$

Now it can be easily generalized to n-mode LC group $\mathcal{C}_2^n$. δ maps n-mode LC operator U to the matrix $Q = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, where A, B, C and D are diagonal matrices, i.e., $A = \text{diag}(a_1, ..., a_n)$, $B = \text{diag}(b_1, ..., b_n)$, $C = \text{diag}(c_1, ..., c_n)$, $D = \text{diag}(d_1, ..., d_n)$, which satisfy that $a_k \times d_k - c_k \times b_k = 1, (k = 1, \cdots, n)$. The fact means that $\begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} (k = 1, \cdots, n)$ is the LC operator on the kth mode.

Matrix representation of CV stabilizer state

A stabilizer which consists of n commutable and independent generators can be represented by a $2n \times n$ matrix Θ, whose columns are the vectors mapped from the Pauli generator operator. Once the matrix Θ is given, the generators S can be determined up to some phase factors, then the stabilize state can be determined.

The sufficient and necessary condition of equivalence between two stabilizer states

Applying the matrix representation of LC group, the evolution of a stabilizer under LC operators can be calculated as $\Theta_2 = Q\Theta_1$.

The stabilizer Θ_1 and Θ_2 are equivalent under LC operation if and only if there exist Q and N satisfying that

$$\Theta_2 = Q\Theta_1 N, \quad (9)$$

where N is a $2n \times 2n$ invertible matrix standing for a elementary column transformation and Q stands for the LC operations.

LC equivalence between stabilizer states and weighted graph states for CV

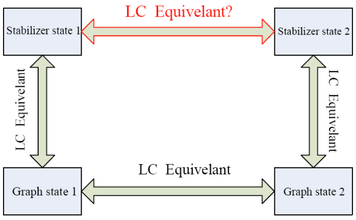
A weighted graph state is described by a mathematical graph $G = (V, E)$, i.e., a finite set of n vertices V connected by a set of edges E, where every edge is specified by a factor Ω_{ab} corresponding to the interaction strength between the modes a and b.

$$g_a = (\hat{p}_a - \sum \Omega_{ab}\hat{x}_b) \to 0 \quad (10)$$

Corresponding stabilizer:

$$\{G_a(\xi) = \exp(-i\xi g_a) = X_a(\xi)\prod Z_b(\Omega_{ab}\xi)|\xi \in \mathbb{R}\}. \quad (11)$$

the generator matrix of a CV weighted graph state can be expressed as $\Theta = \begin{bmatrix} G \\ I \end{bmatrix}$, with G a symmetrical matrix standing for the adjacent matrix for the weighted graph and I an n-order identity matrix. Important: A stabilizer state is equivalent to a weighted graph state under LC operations, the detailed proof can be obtained in [1].



Matrix equation to determine the equivalence between stabilizer states under LC operations

The matrix equation determining whether two stabilizer states are equivalent can be expressed as

$$\begin{cases} G_1A + B - G_1CG_2 - DG_2 = 0 \\ AD - BC = I \\ |CG_1 + D| = 0 \end{cases} \quad (12)$$

where A, B, C and D are diagonal matrices.

Application to five-mode unweighted graph states

Applying the above criterion to every pair of five-mode unweighted graph states with different adjacent matrices whose number is 728, we get 28 different graph states that are not LC equivalent, whose graphs are shown in Fig.3. It is obvious that some of the graphs in Fig.3 are isomorphic, but they cannot be transformed into each other without exchanging modes of different vertices. Here No.M(N) represents that there are N graph states that are equivalent with graph state No.M under LC operations. In the case of DV, DV graph states are LC equivalent if and only if the graphs of the graph states are equivalent under local complement operation. However, for the case of CV, this rule is no longer available.

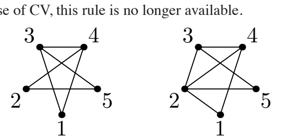


Fig.2 Applying local complement on the node 4 of the graph on the left, one can get the graph on the right. However there does not exist local Clifford operations to implement such transformation.

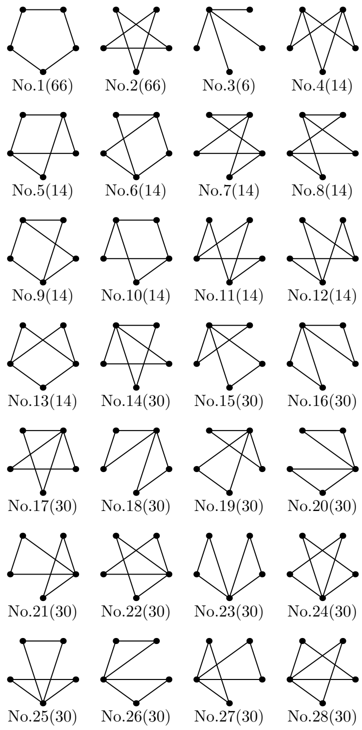


Fig.3 Classification of five-mode unweighted graph states that are not equivalent to each other under local Clifford operators.

Quantum network using CV graph state

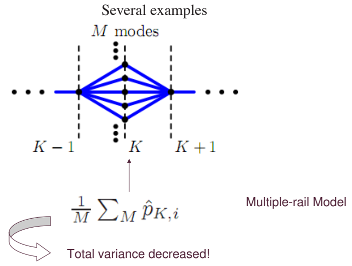
Sufficient conditions for the usefullness of a graph state (characterized by its adjacency matrix G) for teleportation via each of four types of UVT [2, 3]:

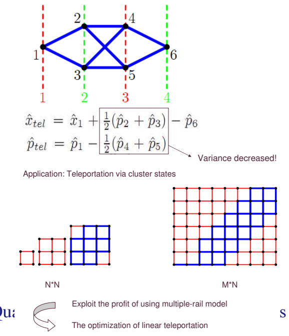
$$\begin{aligned}
&(I) && rank(G^*) = rank(G^* \quad I_1 - I_N) \\
& && \quad\quad = rank(G^* \quad -G_1 - G_N) \\
&(II) && rank(G^*) = rank(G^* \quad I_1 + I_N) \\
& && \quad\quad = rank(G^* \quad -G_1 + G_N) \\
&(III) && rank(G^*) = rank(G^* \quad -G_1 + I_N) \\
& && \quad\quad = rank(G^* \quad I_1 - G_N) \\
&(IV) && rank(G^*) = rank(G^* \quad -G_1 - I_N) \\
& && \quad\quad = rank(G^* \quad I_1 + G_N)
\end{aligned} \quad (13)$$

Quantum network communication results:

$$\begin{aligned}
(I,III) & \begin{cases} \hat{x}_{tel} = \hat{x}_1 + \sum_{i=2}^{N-1}\alpha_i\hat{p}_i \\ \hat{p}_{tel} = \hat{p}_1 + \sum_{i=2}^{N-1}\beta_i\hat{p}_i \pm \hat{p}_N \end{cases} \\
(II,IV) & \begin{cases} \hat{x}_{tel} = \hat{x}_1 + \sum_{i=2}^{N-1}\beta_i\hat{p}_i \pm \hat{p}_N \\ \hat{p}_{tel} = \hat{p}_1 + \sum_{i=2}^{N-1}\alpha_i\hat{p}_i \end{cases}
\end{aligned} \quad (14)$$

Several examples





A new measurement method of entanglement degree of CV entangled state

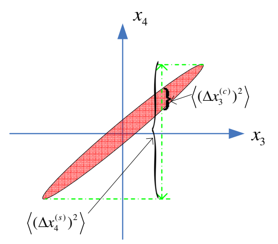
Parameter R is defined as the ratio of single-mode and coincidence widths of distributions which can be found from single-mode and coincidence measurements,

$$R_x = \frac{\Delta x_{1,2}^{(s)}}{\Delta x_{1,2}^{(c)}} \quad \text{or} \quad R_y = \frac{\Delta y_{1,2}^{(s)}}{\Delta y_{1,2}^{(c)}} \quad (15)$$

where Δx, Δy are the quadrature amplitude and phase wave packet widths, superscript (s) and (c) refer to single-mode measurement and coincidence measurement, subscript (1,2) refer to models 1 and 2.

$$\frac{d\omega^{(uncond)}(x_1)}{dx_1} = \int dx_2|\Psi(x_1, x_2)|^2, \quad (16)$$

$$\frac{d\omega^{(cond)}(x_1)}{dx_1} = \frac{|\Psi(x_1, x_2)|^2}{\int dx_1|\Psi(x_1, x_2)|^2}|_{x_2=constant}. \quad (17)$$



The parameter R of CV entanglement pairs generated by NOPA is obtained

$$R = \frac{\Delta x_i^{(s)}}{\Delta x_i^{(c)}} = \frac{\Delta y_i^{(s)}}{\Delta y_i^{(c)}} = \cosh 2r, \text{ for } i = 3, 4. \quad (18)$$

The calculation shows that [4] the parameter R is same as both Schmidt number K and parameter F related with Reid's method.

References

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