Comment on "Teleportation of two-mode squeezed states"

Guangqiang He* and Jingtao Zhang
State Key Lab of Advanced Optical Communication Systems and Networks Department of Electronic Engineering, Shanghai Jiaotong University, Shanghai 200030, China

(Received 24 April 2011; published 24 October 2011)

We investigate the teleportation scheme of two-mode squeezed states proposed by Adhikari *et al.* [S. Adhikari *et al.*, Phys. Rev. A **77**, 012337 (2008)]. It uses four-mode entangled states to teleport two-mode squeezed states. The fidelity between the original two-mode squeezed states and teleported ones is calculated. The maximal fidelity value of Adhikari's protocol is 0.38, which is incompatible with the fidelity definition with the maximal value 1. In our opinion, one reason is that they calculate the fidelity for multimodes Gaussian states using the fidelity formula for single-mode ones. Another reason is that the covariance matrix of output states should be what is obtained after applying the linear unitary Bogoliubov operations (two cascaded Fourier transformations) on the covariance matrix given in Eq. (12) in their paper. These two reasons result in the incomparable results. In addition, Adhikari's protocol can be simplified to be easily implemented.

DOI: 10.1103/PhysRevA.84.046301 PACS number(s): 03.67.Hk, 03.67.Mn, 42.50.Dv

It is well known that the Uhlmann fidelity between a pure state ρ_1 and a mixed one ρ_2 reduces to $\mathrm{Tr}(\rho_1\rho_2)$. If both states were undisplaced Gaussian ones the fidelity is given by

$$F(\rho_1,\rho_2) = \frac{1}{\sqrt{\det\frac{A_1+A_2}{2}}}. \quad (1)$$

Here A_1 and A_2 represent the covariance matrix of the input states and output states, respectively. And in this Comment, the parameters q, r, k, c, s, η, ϕ are defined in Ref. [1].

To obtain the exact expression of fidelity between input states and teleported states, the input and output covariance matrix should be substituted into Eq. (1). The input covariance matrix is shown by Eq. (5) in Ref. [1]. However $\sigma^{(13)(14)}$ in Eq. (12) in Ref. [1] is not the appropriate covariance matrix of the output state since for $k=-1$ and $r\to\infty$ one obtains $\sigma^{(13)(14)} = (\sigma')^{(7)(8)}$ which is different from the input state represented by $\sigma^{(7)(8)}$. Though Adhikari *et al.* have pointed out that $(\sigma')^{(7)(8)}$ is equivalent to $\sigma^{(7)(8)}$ under local linear unitary Bogoliubov operations (LLUBOs) [2], their protocol is incomplete since the output state of teleportation should be as similar to the input state as possible. In fact, it is easy to see that

$$S_F\sigma^{(13)(14)}S_F^T = \sigma^{(7)(8)} + 2(c+ks)I \quad (2)$$

is valid for either $S_F = \mathrm{diag}(1,1,-1,-1)$ or $S_F = \mathrm{diag}(-1,-1,1,1)$. We define the covariance matrix of teleported modes as

$$\sigma_{\mathrm{tel}} = S_F\sigma^{(13)(14)}S_F^T = \sigma^{(7)(8)} + 2(c+ks)I \quad (3)$$

and Eq. (3) means that by applying Fourier transformation twice on either mode 13 or 14 twice one will get a state similar to the input state. For $k=-1$ and $r\to\infty$ one obtains $\sigma_{\mathrm{tel}} = \sigma^{(7)(8)}$. Substituting Eq. (5) in Ref. [1] and Eq. (3) into Eq. (1) one can get the expression for fidelity as follows

$$F(\rho_1,\rho_2) = \frac{1}{M+N}, \quad (4)$$

where $M = 2[\cosh(2q)+\cosh(2r)][\cosh(2r)+\sin(2\phi)\sinh(2r)]$, $N = -[\cos(2\phi)]^2[\sinh(2r)]^2$.

It is difficult to estimate the fidelity for such a complex expression. Here we setup the phase of amplifiers as $\phi = -\frac{\pi}{4}$ without loss of generality, then the fidelity reduces into

$$F(\rho_1,\rho_2) = \frac{1}{2[\cosh(2q)+\cosh(2r)]e^{-2r}}. \quad (5)$$

The fidelity $F(\rho_1,\rho_2)$ in terms of the squeezing factors r,q is depicted in Fig. 1. Obviously, the fidelity is a monotonic increasing function with respect to the squeezing factor r, meaning that the better the quantum channel is, then the better performance the teleportation has. However, the fidelity is a monotonic decreasing function with respect to the parameter of a two-mode squeezed state to be teleported, meaning that it is more difficult to teleport better entangled states. When $q=0$ (i.e., the teleported state consists of two vacuum states), the maximum value of fidelity can be obtained with r tending to infinity

$$\lim_{|r|\to\infty} F(\rho_1,\rho_2)|_{q=0} = 1. \quad (6)$$

Note that the formula for the fidelity used in Ref. [1] is valid only in the one-mode Gaussian case. So we present the general expression of fidelity [Eq. (4)] whose maximum value is comparable with the standard definition of fidelity given by Uhlmann [3].

Furthermore, we consider the beam splitters BS1 and BS2 in Ref. [1] as not necessary. In fact, one can use the product of two identical two-mode entangled states to implement the protocol without loss in both the fidelity between the input and the output states and in the entanglement degree of the output states. To see this, notice that $\sigma^{(5)(6)(15)(16)}$ in Eq. (4) in Ref. [1] is the covariance matrix of the output of beam splitters 1 and 2, and Eq. (8) in Ref. [1] represents the rest physical process (the beam splitters used by Alice, the measurements performed by Alice, and the unitary transformation performed by Bob). However, it turns out that replacing $\sigma^{(5)(6)(15)(16)(7)(8)}$ in Eq. (8) in Ref. [1] using $\sigma^{(1)(3)(2)(4)}\bigoplus\sigma^{(7)(8)}$ one will get Eq. (10) in Ref. [1] as well. That is to say, Bob supplies modes 1 and 2 to Alice and keeps modes 3 and 4. Alice combines modes 1 and

*gqhe@sjtu.edu.cn

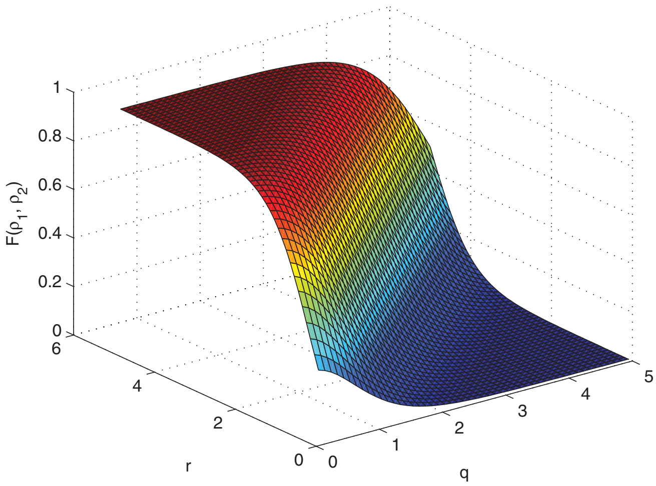

FIG. 1. (Color online) The fidelity of teleportation F with respect to both the parameter of quantum channel r and the parameter of teleported two-mode squeezed states q.

7 with BS3 to get modes 9 and 10, and modes 2 and 8 with BS4 to get modes 11 and 12. And then Alice measures these modes (9, 10, 11, 12) to get the results $(X_9, P_{10}, X_{11}, P_{12})$, which she communicates to Bob. Bob then uses the measurement results to displace the state of modes 3 and 4 by applying the unitary transform in Eq. (7) in Ref. [1], and he finally will get the same output as in Ref. [1]. So beam splitters 1 and 2 in Ref. [1] will not improve the performance of teleportation in the case that there is no additional noise from the environment. The advantage of using these two beam splitters is not obvious. Thus the scheme can be simplified.

In conclusion, though the scheme suggested by Adhikari *et al.* can realize the teleportation of two-mode squeezed states, they failed to give the correct expression for the fidelity of the teleportation, and their protocol is too complicated so there may be some simplification of it.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grants No. 61102053, No. 61170228, No. 60970109, and No. 60801051) and SJTU PRP (Grants No. T030PRP18001 and No. T030PRP19035).

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