

Controlled teleportation of continuous variables

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Received 13 May 2008, in final form 18 July 2008

Published 26 August 2008

Online at stacks.iop.org/JPhysB/41/175504

Abstract

A controlled quantum teleportation of continuous variable using a tripartite Greenberger–Horne–Zeilinger (GHZ) state is proposed. The protocol enables controlling the fidelity of bipartite teleportation by shifting the phase of one of the three modes involved in the teleportation. This method reduces the quality of the tripartite entanglement and by quadrature measurements of the changed mode, bipartite entanglement can be distilled from the original entanglement. Because of the phase shift, the quality of the bipartite entanglement is inferior to the entanglement distilled from the original GHZ-state. By controlling the quality of the entanglement we realize the controlled teleportation.

1. Introduction

Quantum teleportation is a process to transmit the complete information of a quantum state by sending classical information with the help of entanglement states. Since Bennett *et al* proposed the initial protocol [1] in 1993, quantum teleportation has attracted much attention, and has been investigated by many groups. Teleportation between two parties, which has been experimentally realized for qubits [2, 3], relies on a bipartite Bell-state entanglement. Teleportation between three parties [4] using a Greenberger–Horne–Zeilinger (GHZ) state needs the help of multipartite entanglement state, and can only teleport the input unknown state to only one of the other two parties because of the no-cloning theorems [5]. Recently, controlled teleportation of a single-qubit [6] and m-qubit [7] has been studied. In these teleportation protocols, the qubits can be regenerated by one of the receivers with the help of the others. In other words, the controller can decide whether the receiver can or cannot obtain the teleported qubits.

Compared with discrete variables, continuous variables [8] are superior because of their simplicity and high efficiency when measuring and manipulating continuous quadratures. For continuous variables, a quantum teleportation protocol has also been proposed [9], and continuous variable teleportation has been experimentally realized [10, 11]. Tripartite teleportation of continuous variables has already been theoretically demonstrated [12] using continuous variable GHZ-state. Recently tripartite teleportation based on the

scheme in [12] has also been experimentally realized [13]. Moreover, a multipartite teleportation network has also been demonstrated both with a multipartite GHZ state [12] and with a continuous variable cluster-state network [14]. Yet the controlled teleportation of continuous variables has not been proposed. In this paper, we will discuss the controlled teleportation of continuous variables and strengthen the control method. A traditional control method will only be able to control the success or failure, whereas ours will make the teleported mode accurately controlled. On the other hand, a recent study in [15] uses a similar method discussing the highest fidelity, when the third party uses different measurement operators to assist the teleportation. Despite the similarity in the method, the difference exists in aspects including success probability (considering their dichotomic measurement), scheme completeness (the operator $\hat{D}(\gamma')$ by Bob is not given in their scheme) and experimental flexibility (they need to change the squeezing parameter). Moreover, our scheme shows superiority in the simple calculation as well as in the wider range of fidelity that we can achieve.

It is known that single-mode squeezed states incident on a beam splitter will yield a GHZ state [16]. Multipartite quantum protocols were proposed theoretically and realized experimentally such as quantum teleportation networks [12] and controlled dense coding [17, 18] based on tripartite entanglement. In all of these protocols, the entanglement plays a key role, and will decide the quality of the outcome. The quality of the entanglement, which is related to the squeezing parameters, will largely decide the fidelity of the

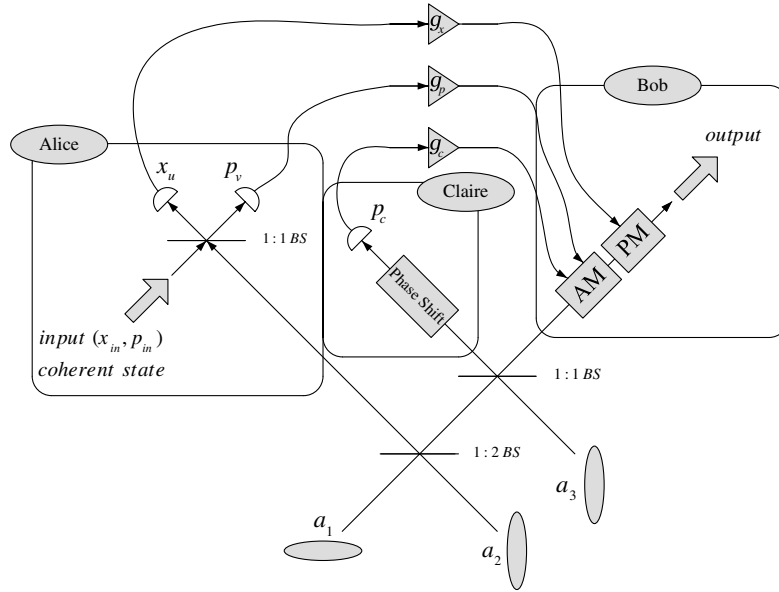


Figure 1. The tripartite quantum teleportation from Alice to Bob under the control of Claire. Three squeezed vacuum states are combined at two beam splitters (BS) with relative phases locked. The three outputs of the two beam splitters are in a tripartite GHZ state, and are sent to the three parties Alice, Bob and Claire, respectively. Symbols and abbreviations are defined in the text.

teleportation. Without the entanglement the best mean fidelity of the reconstructed mode is $F = \frac{1}{2}$ [19]. By controlling the squeezing parameter of the quantum entanglement, we can control the possible optimum fidelity of the teleportation.

Although in most cases we hope that the output mode can be as close to the unknown input mode as possible, we sometimes also hope to limit the best fidelity and make the output not so close to the initial input mode, because the receiver may not be trusted. Quantum secret sharing (QSS) [20] is an important application of the controlled teleportation, as it has been discussed in [21]. Since our scheme is superior, it is fully capable of implementing QSS. We can also distribute a secret message using our scheme. Assume that Alice needs to distribute a secret message to many receivers under the supervision of Claire by sending identical modes. But the receivers may not be equally qualified or trusted. Some of the receivers may need to know the details of it while others do not. But experimentally it is hard to control the fidelity by changing the squeezing parameters. When the entanglement they are using is predetermined, we even have no method to realize the control. To solve this problem, we consider controlling the quality of the multipartite entanglement by some modulations of Claire. In this paper, we propose an easy way to get the desirable teleportation fidelity.

The paper is outlined as follows. In section 2, we propose our protocol of the controlled bipartite teleportation as well as the strengthened control method. In section 3, we compute the fidelity of the teleported mode after the control, and make some analysis of our protocol according to the result. Finally, in section 4, we present our conclusions.

2. Controlled continuous variable bipartite teleportation using a GHZ state

The proposed scheme we discuss in this paper can be employed to control the quality of the teleportation measured

by the optimum fidelity. The ‘position’ and ‘momentum’ are the canonical quantum quadratures of a single-mode electromagnetic field defined as $\hat{x} = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$ and $\hat{p} = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$. Thus, \hat{x} and \hat{p} obey the Heisenberg uncertainty relation $\Delta\hat{x}\Delta\hat{p} \geq \frac{1}{4}$. We assume that Alice wants to send an unknown mode to Bob, while Claire supervises and controls the transmission as well as its quality through her measurement. The protocol can be described generally by the following steps (see figure 1).

Step 1. Preparation of a tripartite GHZ state. Create three squeezed modes (one momentum-squeezed mode and two position-squeezed modes) from the initial vacuum modes with a common squeezing parameter r . Then a tripartite GHZ state can be generated from the squeezed modes by combining them to the beam splitters of 1:2 and 1:1. Then we send three outputs to Alice, Bob and Claire, respectively.

Step 2. Alice performs a joint measurement or so-called ‘Bell measurement’ on her entangled mode and an unknown input mode by coupling the two modes to a 1:1 beam splitter. Then she measures the classical values of \hat{x}_u and \hat{p}_v and reduces them to x_u and p_v , and then sends them to Bob.

Step 3. Assume Claire wants the teleportation between Alice and Bob to be restricted to a certain degree between the success floor and failure floor when squeezing parameter r has been determined previously. Claire first proposes the phase shift (which will be defined later) on her mode and then measures the classical value of the momentum of changed mode, and sends the result to Bob via classical channels.

Step 4. Bob can now accomplish the teleportation with the information from Alice and Claire by displacement operations on his mode on both \hat{x} and \hat{p} . After the operations, Bob finally gets the output of the teleported output.

Now let us further explain the steps stated above. We define the action of an arbitrary beam splitter operation on a pair of modes i and j as

$$\hat{B}_{ij}(\eta) : \begin{cases} \hat{a}_i \rightarrow \hat{a}_i \sqrt{\eta} + \hat{a}_j \sqrt{1-\eta}, \\ \hat{a}_j \rightarrow \hat{a}_i \sqrt{1-\eta} - \hat{a}_j \sqrt{\eta}. \end{cases} \quad (1)$$

Applying the beam splitters of 1:2 and 1:1 respectively to the momentum-squeezed mode 1 and position-squeezed modes 2 and 3 (see figure 1) yields a GHZ state, with the squeezing parameters of all states being r . The tripartite GHZ state is expressed in Heisenberg operators:

$$\begin{aligned} \hat{x}_1 &= \frac{1}{\sqrt{3}}e^{+r}\hat{x}_1^{(0)} + \sqrt{\frac{2}{3}}e^{-r}\hat{x}_2^{(0)}, \\ \hat{p}_1 &= \frac{1}{\sqrt{3}}e^{-r}\hat{p}_1^{(0)} + \sqrt{\frac{2}{3}}e^{+r}\hat{p}_2^{(0)}, \\ \hat{x}_2 &= \frac{1}{\sqrt{3}}e^{+r}\hat{x}_1^{(0)} - \frac{1}{\sqrt{6}}e^{-r}\hat{x}_2^{(0)} + \frac{1}{\sqrt{2}}e^{-r}\hat{x}_3^{(0)}, \\ \hat{p}_2 &= \frac{1}{\sqrt{3}}e^{-r}\hat{p}_1^{(0)} - \frac{1}{\sqrt{6}}e^{+r}\hat{p}_2^{(0)} + \frac{1}{\sqrt{2}}e^{+r}\hat{p}_3^{(0)}, \\ \hat{x}_3 &= \frac{1}{\sqrt{3}}e^{+r}\hat{x}_1^{(0)} - \frac{1}{\sqrt{6}}e^{-r}\hat{x}_2^{(0)} - \frac{1}{\sqrt{2}}e^{-r}\hat{x}_3^{(0)}, \\ \hat{p}_3 &= \frac{1}{\sqrt{3}}e^{-r}\hat{p}_1^{(0)} - \frac{1}{\sqrt{6}}e^{+r}\hat{p}_2^{(0)} - \frac{1}{\sqrt{2}}e^{+r}\hat{p}_3^{(0)}, \end{aligned} \quad (2)$$

where the superscript '(0)' denotes the initial vacuum modes. This GHZ state is an eigenstate of total momenta $p_1+p_2+p_3 \rightarrow 0$ with relative positions $x_i - x_j \rightarrow 0$ ($i, j = 1, 2, 3$), when $r \rightarrow \infty$. We send the three modes of equations (2) to Alice, Bob and Claire, respectively. Alice wants to teleport an unknown quantum state $\hat{x}_{in}, \hat{p}_{in}$ to Bob. She performs a joint measurement by first coupling her mode 1 with the unknown mode using the 1:1 beam splitter, and expresses as $\hat{x}_u = (\hat{x}_{in} - \hat{x}_1)/\sqrt{2}$, $\hat{p}_v = (\hat{p}_{in} + \hat{p}_1)/\sqrt{2}$. Then, Alice measures \hat{x}_u and \hat{p}_v and sends the classical measurement result x_u and p_v to Bob via classical channels.

Let us rewrite Bob's mode 2 as

$$\begin{aligned} \hat{x}_2 &= \hat{x}_{in} - (\hat{x}_1 - \hat{x}_2) - \sqrt{2}\hat{x}_u, \\ \hat{p}_2 &= \hat{p}_{in} + (\hat{p}_1 + \hat{p}_2 + g_c\hat{p}_3) - \sqrt{2}\hat{p}_v - g_c\hat{p}_3, \end{aligned} \quad (3)$$

where g_c is the gain determined later by the measurement of \hat{p}_3 . Here we assume an arbitrary coherent-state input $|\alpha_{in}\rangle = |x_{in} + ip_{in}\rangle$, and the fidelity is defined by $F \equiv \langle \alpha_{in} | \hat{\rho}_{tel} | \alpha_{in} \rangle$ [19]. In the case of the gain $g_x = g_p = \sqrt{2}$ (see figure 1), the fidelity for the single-mode Gaussian states is given by $F = 2/\sqrt{(1+4\langle \delta^2 \hat{x}_{tel} \rangle)(1+4\langle \delta^2 \hat{p}_{tel} \rangle)}$. Now Bob can reconstitute the input state provided that Claire sends the classical value of the momentum of mode 3 to him. Claire can choose either to send the measurement result to Bob, who can achieve a perfect teleportation as $r \rightarrow \infty$, or not to send the result to Bob, who has the teleported mode $\hat{x}_{tel} = \hat{x}_{in} - (\hat{x}_1 - \hat{x}_2)$, $\hat{p}_{tel} = \hat{p}_{in} + (\hat{p}_1 + \hat{p}_2)$ after the modulation of mode 2 according to the given information from Alice ($\hat{x}_2 \rightarrow \hat{x}_{tel} = \hat{x}_2 + \sqrt{2}x_u$, $\hat{p}_2 \rightarrow \hat{p}_{tel} = \hat{p}_2 + \sqrt{2}p_v$). In this case, Bob can only get the fidelity of $F = [(1+e^{-2r})(1+2e^{-2r}/3+e^{+2r}/3)]^{-1/2}$. When $r \rightarrow \infty$, $F \rightarrow 0$. We assume that the previous result means the teleportation succeeds, and the latter result fails (although the fidelity above $\frac{1}{2}$ can still be achieved by choosing certain r). Now Claire can control

the success or the failure of the teleportation by deciding whether to send her classical result or not. However, the values of F between success and failure cannot be achieved under certain r .

In order to control the quality of the teleportation from Alice to Bob with predetermined r , we define the action of an ideal phase shift (see figure 1) operation on one mode as

$$\hat{P}_i(\theta) : \begin{cases} \hat{x}_i \rightarrow \hat{x}_i \cos \theta + \hat{p}_i \sin \theta, \\ \hat{p}_i \rightarrow \hat{p}_i \cos \theta - \hat{x}_i \sin \theta. \end{cases} \quad (4)$$

It can change the phase of the mode to an arbitrary degree. Claire proposes the phase shift in equations (4) on her mode 3 and changes \hat{x}_3, \hat{p}_3 to \hat{x}'_3, \hat{p}'_3 . Claire measures the momentum of the changed mode \hat{p}'_3 and reduces it to p'_3 , and then sends the result to Bob via classical channels.

Since the phase change on mode 3 will not alter the expression of mode 2 (although this will certainly affect the property of the entanglement), we can still rewrite Bob's mode 2 as equations (3). Bob can now accomplish the teleportation with the information from Alice and Claire by the action $\hat{x}_2 \rightarrow \hat{x}_{tel} = \hat{x}_2 + \sqrt{2}x_u$, $\hat{p}_2 \rightarrow \hat{p}_{tel} = \hat{p}_2 + \sqrt{2}p_v + g_c p'_3$. Bob can realize the modulation by using amplitude and phase modulators (AM and PM in figure 1). Now Bob gets the teleported mode, expressed as

$$\begin{aligned} \hat{x}_{tel} &= \hat{x}_{in} - \sqrt{\frac{3}{2}}e^{-r}\hat{x}_2^{(0)} + \frac{1}{\sqrt{2}}e^{-r}\hat{x}_3^{(0)}, \\ \hat{p}_{tel} &= \hat{p}_{in} - \frac{g_c \sin \theta}{\sqrt{3}}e^{+r}\hat{x}_1^{(0)} + \frac{(2+g_c \cos \theta)}{\sqrt{3}}e^{-r}\hat{p}_1^{(0)} \\ &\quad + \frac{g_c \sin \theta}{\sqrt{6}}e^{-r}\hat{x}_2^{(0)} + \frac{(1-g_c \cos \theta)}{\sqrt{6}}e^{+r}\hat{p}_2^{(0)} \\ &\quad + \frac{g_c \sin \theta}{\sqrt{2}}e^{-r}\hat{x}_3^{(0)} + \frac{(1-g_c \cos \theta)}{\sqrt{2}}e^{+r}\hat{p}_3^{(0)}. \end{aligned} \quad (5)$$

When $\theta = 0$, the teleported mode becomes

$$\begin{aligned} \hat{x}_{tel} &= \hat{x}_{in} - (\sqrt{3}e^{-r}\hat{x}_2^{(0)} - e^{-r}\hat{x}_3^{(0)})/\sqrt{2}, \\ \hat{p}_{tel} &= \hat{p}_{in} + (2+g_c)e^{-r}\hat{p}_1^{(0)}/\sqrt{3} + (1-g_c)e^{+r}\hat{p}_2^{(0)}/\sqrt{6} \\ &\quad + (1-g_c)e^{+r}\hat{p}_3^{(0)}/\sqrt{2}. \end{aligned} \quad (6)$$

The teleported mode in equations (6) is the same with the result discussed in [12], because no difference is made by the operation in equations (4) when $\theta = 0$. When $r \rightarrow \infty$ and $\theta = 0$, from equations (5) we can easily see that perfect teleportation can be achieved as $\hat{x}_{tel} = \hat{x}_{in}$, $\hat{p}_{tel} = \hat{p}_{in}$.

3. Fidelity measures

Let us now assess how well one can generally succeed in teleporting the state by computing the fidelity of our teleportation scheme. The definition of fidelity has been given in section 2. In the case of the gain $g_x = g_p = \sqrt{2}$, with $g_c = [2 \cos \theta (e^{+4r} - 1)] / [(1 + \cos^2 \theta) e^{+4r} + (1 + \sin^2 \theta)]$ the optimum teleportation fidelity can be achieved:

$$\begin{aligned} F_{opt} &= [1 + e^{-2r}]^{-\frac{1}{2}} \\ &\quad \times \left[1 + \frac{\sin^2 \theta e^{+4r} + 4 \sin^2 \theta e^{-4r} + 5 \cos^2 \theta + 4}{3(1 + \cos^2 \theta) e^{+2r} + 3(1 + \sin^2 \theta) e^{-2r}} \right]^{-\frac{1}{2}}. \end{aligned} \quad (7)$$

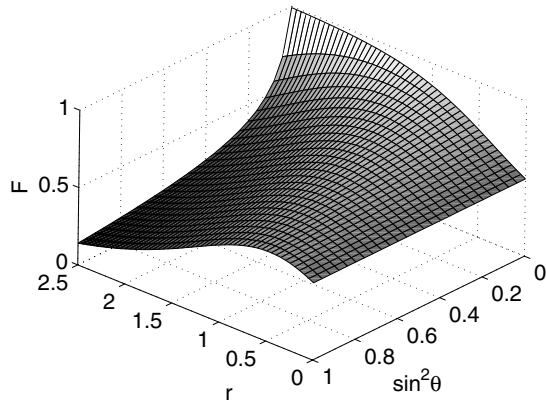


Figure 2. Optimized fidelity for the teleportation from Alice to Bob with different θ chosen by Claire.

Figure 2 shows the fidelity of equation (7) for the squeezing parameter r .

For $\theta = 0$, we obtain the same fidelity of $F_{\text{opt}} = \{[1 + e^{-2r}][1 + 3/(2e^{+2r} + e^{-2r})]\}^{-1/2}$ as calculated in [12], which is the optimum fidelity under certain r . For $\theta = \pi/2$, when the measurement of \hat{p}'_3 contains no information about the original \hat{p}_3 , the optimum fidelity $F_{\text{opt}} = \{[1 + e^{-2r}][1 + (e^{+4r} + 4e^{-4r} + 4)/(6e^{+2r} + 3e^{-2r})]\}^{-1/2}$, which is the worst for certain r (lower than the failure case of teleportation which computed in section 2). Any fidelity values between the two floors can be achieved by the choice of θ . In fact, when r is determined previously and Claire chooses $0 < \theta < \pi/2$, the fidelity of the teleportations F_{opt} and θ is monotonous for arbitrary r . She can let Bob get the teleported mode with whatever fidelity she wants between the two floors. Controlling the fidelity of the bipartite teleportation is therefore realized. However, as the squeezing parameters become greater (or the quality of the entanglement is better), the choice of θ has to be more accurate. When the original beams are highly squeezed, a very small shift of θ will greatly influence the fidelity that Bob can achieve. Figure 3 shows the fidelity of equation (7) for $\sin^2 \theta$ under different squeezing parameters.

It is also needed to be pointed out that for any $\theta \neq 0$, $F_{\text{opt}} \rightarrow 0$ when $r \rightarrow \infty$. That indicates, the optimum fidelity is not achieved when the original vacuum mode is infinitely squeezed (except when $\theta = 0$). Fidelity is a measurement of the entanglement between Alice and Bob. For infinite squeezing, the entanglement properties of the GHZ state are easily destroyed if a part of the system is disturbed. In our scheme, the modes 1 and 2 are completely unentangled after a slight change in the mode 3, which conforms the conclusion in [22]. In fact, for both position and momentum, the infinite squeezing provides a zero uncertainty on one quadrature, and at the same time an infinite uncertainty on the other because of the Heisenberg uncertainty. A shift of phase, no matter how small, will introduce an infinite noise to the teleported mode, and thus will make $F = 0$. However, for arbitrary θ , there is a corresponding r to achieve the highest fidelity. Figure 4 shows the relationship for squeezing parameter r .

Interestingly, when $\theta = \pi/2$, \hat{p}'_3 contains no information of original \hat{p}_3 , but the fidelity is still related to the squeezing

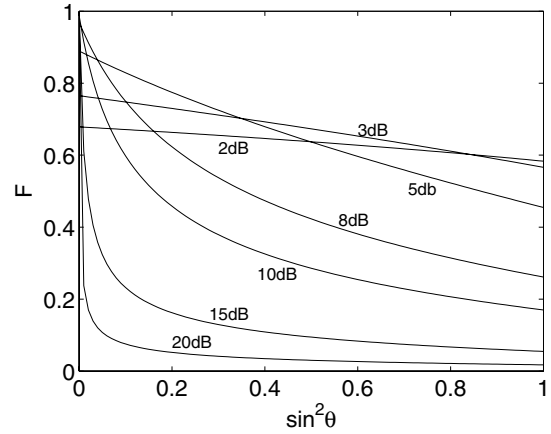


Figure 3. This figure shows the optimum fidelity for the changing θ under certain squeezing (2 dB, 3 dB, 5 dB, 8 dB, 10 dB, 15 dB, 20 dB). The gradient $\partial F/\partial \sin^2 \theta$ becomes greater as the squeezing parameter grows. The more the original vacuum states are squeezed, the more accurate θ is required to achieve a certain fidelity.

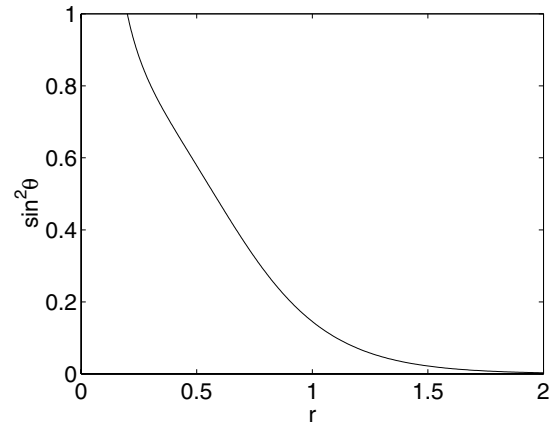


Figure 4. This figure shows the optimum choice of the squeezing parameter when θ is changing. When $\theta \rightarrow 0$, the optimum choice is infinite squeezing. Otherwise, the infinite squeezing will cause such large noise as to undermine the unknown input state.

parameter. The best fidelity $F_{\text{opt}} = 0.66 > 0.5$ is achieved when $r = 0.77(3.33 \text{ dB})$. An average fidelity $F_{\text{av}} > \frac{1}{2}$ can only be achieved using the entanglement [19]. That is to say, Alice and Bob are still entangled without the help of Claire. Actually, the function of measuring the mode 3 and informing the result to Bob is to reduce the GHZ-state to a bipartite entanglement. The entanglement is largely determined by the choice of θ .

Now we elaborate on Claire's role in this scheme. Whether Alice and Bob are entangled or not is determined by Claire's measurement. Claire can adjust the value of θ to modulate the entanglement, and consequently the fidelity of the teleportation between Alice and Bob. Claire can also control the success or failure by sending her measurement result or not. It is only after Bob receives Claire's measurement result that he knows about the entangled state between them. By adjusting the value of θ Claire actually increases the noise level of the entanglement, which will make the entanglement worse and cannot achieve the high fidelity before the shift.

In our scheme, the noise level of Claire's mode 3 is relative to the momentum, \hat{p}_3 . There is still another way to realize the control of the fidelity of teleportation. When Claire gets her mode 3, instead of shifting the phase of the mode, she can couple a mode, either squeezed or vacuum, on the mode 3 to increase its noise. Thus, this method can also limit the optimum fidelity of the teleported mode that Bob gets. But for Claire, she needs to generate squeezed modes, which are difficult to create, or vacuum modes in order to control the noise floor. Then, Claire can couple the two modes with a beam splitter. Yet, Claire cannot restrict the fidelity to the exact value she wants unless squeezed light of arbitrary squeezing parameters can be generated, or the beam splitter of arbitrary dividing parameters is available. In contrast, the phase shift can be done without additional light or with too many devices. This shows the superiority of our scheme.

4. Conclusion

We have introduced a scheme to control the optimum fidelity of continuous variable Gaussian state tripartite teleportation. This scheme helps Claire in supervising the teleportation, and controlling the quality of quantum communication more accurately. The quality of the fidelity is measured by the fidelity between the input and output modes. Physically this scheme requires few optical devices and is relatively easy to realize. We make further discussion about the fidelity when the squeezing parameters are different, and when Claire chooses different controlling strategies. Our scheme can also be applied to control multipartite teleportation.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (grant no 60773085), Shanghai Jiaotong University (SJTU) Young Teacher Foundation (grant no. A2831B) and SJTU PRP (grant no T03013002).

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