Performance analysis of quantum access network using code division multiple access model*

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(Received 2 January 2017; revised manuscript received 13 February 2017; published online 20 April 2017)

Quantum access network has been implemented by frequency division multiple access and time division multiple access, while code division multiple access is limited for its difficulty to realize the orthogonality of the code. Recently, the chaotic phase shifters are proposed to guarantee the orthogonality by different chaotic signals and spread the spectral content of the quantum states. In this letter, we propose to implement the code division multiple access quantum network by using chaotic phase shifters and synchronization. Due to the orthogonality of different chaotic phase shifter, every pair of users can faithfully transmit quantum information through a common channel and have little crosstalk between different users. Meanwhile, the broadband spectra of chaotic signals efficiently help the quantum states to defend against the channel loss and noise.

Keywords: quantum access network, chaotic phase shifter, code division multiple access

PACS: 03.67.–a, 42.50.–p, 42.50.Ex **DOI:** 10.1088/1674-1056/26/6/060304

1. Introduction

Quantum access network[1–7] has been gaining increasing interest, since it can provide reliable infrastructure layer for a large number of users. In quantum access network, multiple pairs of nodes can transmit quantum information with proper encoding into and decoding from the quantum states by using multiple access, which permits simultaneous transmission of multiple quantum data via a common channel. Nowadays, the popular methods of multiple access in optical cryptography include frequency division multiple access[4,5] and time division multiple access,[1] while code division multiple access is not applied to enlarge the scale. Although a corresponding keyed code division multiple access (CDMA) in quantum noise scheme[8,9] is proposed to obtain key expansion in the quantum case, it offers the nonorthogonal set of M-ry states and cannot be used to expand the number of users in the quantum network. Recently, the chaotic phase shifter[10,11] has been proved to be a decoherence suppressor and then suppress the influence from the environment to the quantum states. Besides, the chaotic shifters are almost orthogonal and broadband due to the quality of chaotic signal, which perfectly satisfy the requirement of the CDMA network. By using Kerr interactions in whispering gallery mode resonators,[12] we can successful achieve coupling between the information-bearing light and the chaotic light for chaotic phase shifter modulation. Experimental demonstration of group synchrony in a system of chaotic optoelectronic oscillators is also achieved in a four-node optoelectronic network.[13]

In this letter, we propose to implement the CDMA quantum access network by using chaotic phase shifters and synchronization among senders and receivers. We use chaotic phase shifters to modulate the quantum states and then use beamsplitter (BS) multiplexers to get quantum superposition. After a common channel, we use beamsplitter demultiplexers and chaotic synchronization to decode the quantum signals at the receiver. By analysis, we find the proposed CDMA network can faithfully transmit the quantum states in very noisy channels and defend the crosstalk between different nodes.

2. Theoretical model

Motivated by code division multiple access theory, we have showed a schematic diagram of our strategy in Fig. 1.

The quantum information sent by any nodes is first encoded by the chaotic phase shifters CPS_i with the Hamiltonian $\delta_i(t)a_i^\dagger a_i$, while $\delta_i(t)$ is a time-dependent classical chaotic signal, $i=1,2,\ldots,n$. Compared to the information-bearing field a_i, this encoding spreads the spectral content of quantum information across the entire spectrum due to the chaotic signals. After encoding, all of these quantum signals are combined by beamsplitters multiplexer, and then transmitted over the common channel. At the end of the channel, the quantum signal is amplified by a phase-insensitive linear amplifier (LA)

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61475099 and 61102053), the Program of State Key Laboratory of Quantum Optics and Quantum Optics Devices (Grant No. KF201405), the Open Fund of IPOC (BUPT) (Grant No. IPOC2015B004), the Program of State Key Laboratory of Information Security (Grant No. 2016-MS-05).

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to compensate the loss of the multiplexer, demultiplexer and the channel, and then divided to N beam by beamsplitters demultiplexer with the inverse infrastructure to the multiplexer. All decomposed beam can be decoded to get the information using $\mathrm{CPS}_{i'}$ with the effective Hamiltonian $-\delta_i(t)a_i^\dagger a_i$.

Each pair of CPS_i land $\mathrm{CPS}_{i'}$ induces phase shifts $\exp[-\mathrm{i}\theta_i(t)]$ and the inverse phase shifts $\exp[\mathrm{i}\theta_{i'}(t)]$ respectively, where $\theta_i(t)=\int_0^t \delta_i(\tau)\mathrm{d}\tau$ and $\theta_{i'}(t)=\int_0^t \delta_{i'}(\tau)\mathrm{d}\tau$. To maintain the high fidelity of the quantum signals, we should precisely control the process of quantum communication and keep the same parameters and evolutions of the related chaotic phase shifters to guarantee $\delta_i(t)=\delta_{i'}(t)$. However, this precise control is impractical for remote participants because any small deviation in the system can greatly affect the evolution of the chaotic process. Therefore, additional classical channels between senders and receivers are assumed to exist to synchronize each pair of CPS_i and $\mathrm{CPS}_{i'}$ as shown in Fig. 1. $D(\alpha_i)$ is displacement operator which is used to generate coherent state.

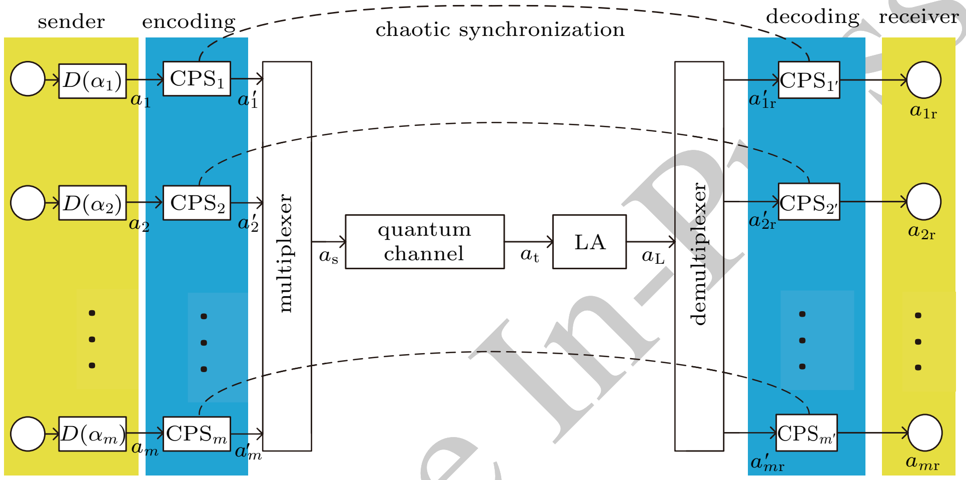


Fig. 1. (color online) Quantum access network scheme using code division multiple access.

We first introduce the structure of multimultiplexer and demultiplexer. We introduce two kinds of multiplexer, corresponding to two kinds of demultiplexer. To achieve the communication between N pairs of users through a common channel, we should build the multiplexer to combine the N pieces of quantum signals. Considering the structure of the BS in the reference,[14] we design the same structure to split and inverse structure to combine the N signals.

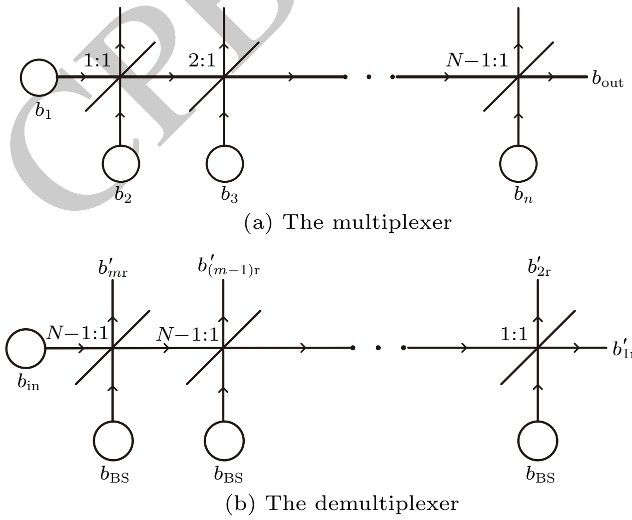


Fig. 2. The multiplexer and demultiplexer by using $N-1$ BS with different transmittance ratio.

Figure 2(a) shows the structure of $N-1$ BS with different transmittance. $b_1, b_2, \ldots, b_n$ are the annihilation operators of the signal fields. It is easy to get the output of the multiplexer is

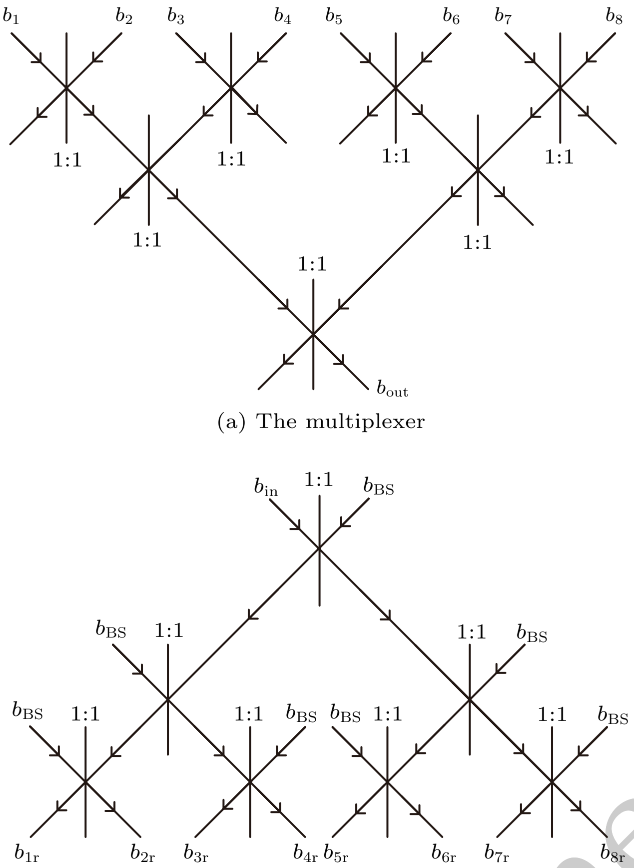
$$b_\mathrm{out}=\sqrt{1/N}\sum_i b_i. \tag{1}$$

And figure 2(b) shows the inverse demultiplexer to help the user to get the same component of the quantum signals from the linear amplifier, where b_BS is the annihilation operator of the vacuum fields entering the beamsplitter, and $b_{1r}', b_2', \ldots, b_n'$ are the annihilation operator of the demultiplexed signal fields. However, it is very difficult to produce the different BS when the number of user is so large. Thus, we give another structure of the multiplexer and demultiplexer.

Figure 3(a) shows the structure of $N-1$ BS with the same transmission. Due to the quality of the binary tree, we require the amount of user to be $2^q, q \in$ integers. Therefore, the output of the multiplexer is

$$b_\mathrm{out}=\sqrt{1/2^q}\sum_i b_i=\sqrt{1/N}\sum_i b_i. \tag{2}$$

Figure 3(b) shows the inverse structure of the binary tree, which aims to get the same component from the source signal, for example, $b_{1r}=\sqrt{1/8}b_\mathrm{in}+(\sqrt{1/8}-1/2-\sqrt{1/2})b_\mathrm{BS}$, $b_{2r}=\sqrt{1/8}b_\mathrm{in}+(\sqrt{1/8}-1/2+\sqrt{1/2})b_\mathrm{BS}$.



(a) The multiplexer

(b) The demultiplexer

Fig. 3. The multiplexer and demultiplexer by using $N-1$ BS with same transmittance ratio.

Then the effect of quantum channel and linear amplifier can be showed as:

$$\begin{aligned} a_{\text{t}} &= \sqrt{T}a_{\text{s}} + \sqrt{1-T}a_0 + \xi_{\text{t}}, \\ a_{\text{L}} &= \sqrt{G}a_{\text{t}} + \sqrt{G-1}a_0^{\dagger} + \xi_{\text{L}}, \end{aligned} \quad (3)$$

where ξ_{t} and ξ_{L} are both a complex random variable with zero mean value and variance ε_{t} and ε_{L} describing additive noise in the channel and the linear amplifier. a_0 is the annihilation operators of the vacuum fields entering the channel, $a_0^{\dagger}$ is the creation operators of the vacuum fields entering the linear amplifier.

Using the chaotic synchronization, the global quantum transmission process of the CDMA network can be described as

$$\begin{aligned} a_{m\text{r}} =& \sqrt{\frac{TG}{N^2}}(a_m + \sum_{n\neq m} a_n\,\mathrm{e}^{\,\mathrm{i}(\theta_m-\theta_n)}) + \mathrm{e}^{\,\mathrm{i}\theta_m}\Bigg(\sqrt{\frac{(1-T)G}{N}}a_0 \\ &+ \sqrt{\frac{G-1}{N}}a_0^{\dagger} + \sqrt{\frac{G}{N}}\xi_{\text{t}} + \sqrt{\frac{1}{N}}\xi_{\text{L}} + \eta a_{\text{BS}}\Bigg), \end{aligned} \quad (4)$$

where $a_{m\text{r}}$ is the m-th received node, a_{BS} is the annihilation operator of the vacuum fields entering the beamsplitter, $|\eta| < 1$ represents the coefficient of the noise added by the BS.

According to the characteristic of the chaotic phase shift $\theta_i(t)$,[10,11] we can take an average over this broadband random signal. We can get the relationship $\overline{\exp(\pm\mathrm{i}\,\theta_i(t))} = \sqrt{M_i}$, and

$$M_i = \exp(-\pi\int_{\omega_{\text{l}i}}^{\omega_{\text{u}i}} \frac{S_{\delta_i}(\omega)}{\omega^2}\,\mathrm{d}\,\omega), \quad (5)$$

where $S_{\delta_i}(\omega)$ is the power spectrum density of signal $\delta_i(t)$, $\omega_{\text{u}i}, \omega_{\text{l}i}$ are the upper bounds and lower bounds of the frequency band of signal $\delta_i(t)$. Then we set $G = N^2/T$, and ignore some excess noise term due to the large number N, equation (4) is further reduced to

$$\begin{aligned} a_{m\text{r}} =& \left(a_m + \sum_{n\neq m}\sqrt{M_m M_n}a_n\right) \\ &+ \sqrt{M_m}\left(\sqrt{\frac{(1-T)N}{T}}a_0 + \sqrt{\frac{N^2-T}{TN}}a_0^{\dagger} + \sqrt{\frac{N}{T}}\xi_{\text{t}}\right). \end{aligned} \quad (6)$$

Considering the broadband frequency spectrum of chaotic signal, all M_i are extremely small,[10] which leads to $a_{m\text{r}} = a_m$. Therefore, faithful transmission of quantum information is successful achieved from the node a_i to node $a_{i'}$. Furthermore, the every different pair of users can set different standard for chaotic phase shifter, which means high safe-level users can have the smaller factor M_i. In the following discussion, we assume all the $M_i = M$.

3. Simulation and analysis

To analyze the quantum network using code division multiple access, we first consider its quantum fidelities. We calculate every node fidelity $F_i = \langle\phi_i|\rho_{i'}|\phi_i\rangle$, where $\rho_{i'}$ is the i-th received quantum sates. According to Eq. (6), the fidelity F_i can be approximated as $F_i = \frac{1}{1+(N-1)M} \approx 1-(N-1)M$. For the chaotic phase shifters, when the Duffing oscillator enters the chaotic regime,[10] the parameter $M \approx 0$, the fidelities $F_i \approx 1$, which means the qubit states can be successfully transmitted through a common channel by using code division multiple access.

Next, we analyze the maximum quantum information transmission rate. The quantum information transmission rate is defined by the quantum capacity,[15–17] and we restrict our discussion to Gaussian channels. The quantum information transmission rate can be showed as[15]

$$\begin{aligned} I(\rho,T) &= H(T[\rho]) - H(\rho,T) \\ &= g(V') - g\left(\frac{D+V'-V_0-1}{2}\right) \\ &\quad - g\left(\frac{D-V'+V_0-1}{2}\right), \end{aligned} \quad (7)$$

where V_0 and V' are the variance of the m-th sent and received state,the parameter D is a variable related to the

$T, G, V_0, V', M, \varepsilon_t$.

$$V' = V_0 + \sum_{n\neq p} M_m M_n V_n + M_m\left(\frac{N^2 - T}{TN} + \frac{N\varepsilon_t}{T}\right),$$
$$D = \sqrt{(V_0 + V' + 1)^2 - 4V_0(V_0 + 1)},$$

and $g(x) = (x+1)\log(x+1) - x\log x$.

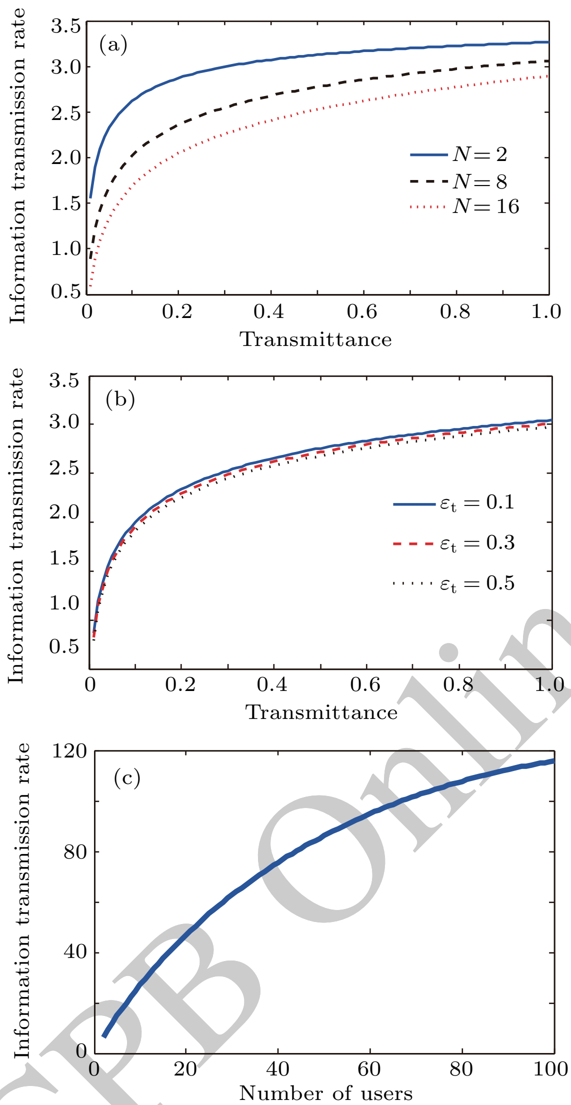


Fig. 4. (color online) Quantum information transmission rates. (a) Quantum information transmission rate under different channel transmittance in same noise. (b) Quantum information transmission rate under different channel transmittance in different noise of 8 pairs of users. (c) Total quantum information transmission rate related to number of users pairs.

First of all, we have showed the information rate under the 2, 8, and 16 pairs of user in Fig. 4(a). We can see the information rate keeps almost same while the transmittance varies in the wide range.When quantum channel is severely lossy, the CDMA network is also a robust network for information transmission. In Fig. 4.(b), we have showed the information rate of 8 pairs of user under the different channel noise $\varepsilon_t = 0.1, 0.3, 0.5$. The information transmission rate is almost not influenced by the channel excess noise, which is the feature of the CDMA technology. In Fig. 4(c), we have showed the relationship between the total information rate and the number of user pairs. We can see information rate increases slower compared to the increase of the user. It is because the linear amplifier vacuum noise term and the channel excess noise term become larger and larger when the number of user pairs increase.

Finally, we are specially concerned about the crosstalk between different nodes for the CDMA network. Here, we calculate the mutual information of different nodes to reflect crosstalk. According to Eq. (6), the mutual information of p-th received node and q-th sent node can be expressed as

$$\begin{aligned} I_{pq} &= \frac{1}{2}\log\frac{V'_p}{V'_{p|q}} \\ &= \frac{1}{2}\log\frac{V_p + \Sigma_{n\neq p} M_p M_n V_n + M_p(\frac{N^2-T}{TN} + \frac{N\varepsilon_t}{T})}{V_p + \Sigma_{n\neq p,q} M_p M_n V_n + M_p(\frac{N^2-T}{TN} + \frac{N\varepsilon_t}{T})} \\ &\approx \frac{1}{2}\left(\frac{M_p M_q V_q}{V_p + \Sigma_{n\neq p,q} M_p M_n V_n + M_p(\frac{N^2-T}{TN} + \frac{N\varepsilon_t}{T})}\right). \end{aligned} \quad (8)$$

Here we use the expression $\log(1+x) \approx x$, for $x \ll 1$.

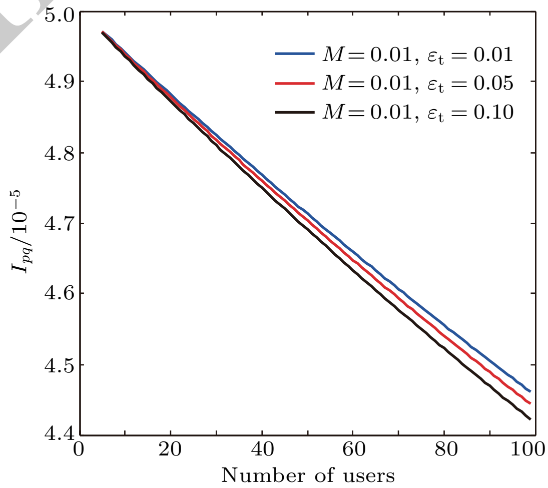


Fig. 5. (color online) The mutual information between p-th received node and q-th sent node under different pairs of users.

In Fig. 5, we have showed the mutual information between different nodes. We can see that the mutual information value is extreme small, which shows crosstalk from the different nodes can be efficiently defended by the chaotic phase shifter due to its orthogonality. When the number of users increase, the single pair mutual information decrease, but it will increase the crosstalk in total. Meanwhile, the mutual information of different nodes is almost not influenced by the noise in the channel.

From these results, we have seen the robustness of the CDMA network using chaotic phase shifter. When the quantum channel is not totally lossy, the information rate of the CDMA network is not obviously influenced by the channel loss. Besides, the excess noise in the channel has almost no effect on the transmission.

Adding the number of user pairs, the total information rate increase while the slope becomes slower. Meanwhile, it can help defend the crosstalk between the different nodes. All of these is mainly due to the character of the code division multiple access. The chaotic phase shifter spread the information-bearing field across a broad spectral band, which help defend the loss and excess noise in the channel to particular mode. In the same time, the non-matched phase shifter will spread other signal as noise and then the crosstalk can be defended, which reveals the orthogonality of different pairs of chaotic phase shifter. However, when the number of user pairs increase, the linear amplifier parameter G should be large enough to compensate the loss of multiplexer and demultiplexer, which will bring a large amount of vacuum noise and decrease the information transmission rate for single pair, although the chaotic phase shifter also decrease the vacuum noise of amplifier. If there exist the non-loss multiplexer and demultiplexer to get and split the superposition of quantum states, the CDMA network can have a better performance in all aspects, especially in the number of users.

4. Conclusion

We have introduced the quantum access network using code division multiple access. Based on the chaotic phase shifter and its synchronization, we can build the CDMA network to achieve the faithful quantum transmission with high fidelity. We thoroughly analyze the CDMA network, finding that it is robust against channel loss, excess noises and crosstalk between different users. Meanwhile, we point out the function of the chaotic phase shifters in the CDMA network by its orthogonality and broadband spectra.

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