The dependence of fidelity on the squeezing parameter in teleportation of the squeezed coherent states*

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This paper investigates an analytical expression of teleportation fidelity in the teleportation scheme of a single mode of electromagnetic field. The fidelity between the original squeezed coherent state and the teleported one is expressed in terms of the squeezing parameter r and the quantum channel parameter (two-mode squeezed state) p. The results of analysis show that the fidelity increases with the increase of the quantum channel parameter p, while the fidelity decreases with the increase of the squeezing parameter r of the squeezed state. Thus the coherent state ($r = 0$) is the best quantum signal for continuous variable quantum teleportation once the quantum channel is built.

Keywords: quantum teleportation, the squeezed coherent states, fidelity

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1. Introduction

Quantum teleportation is the disembodied transport of an unknown quantum state from the sender to receiver using both quantum correlation called entanglement and classical communication.[1,2] It has been considered very important in quantum computation and quantum information.[3,4] Since Bennett *et al.* proposed quantum teleportation which transports an unknown state of any discrete variable (DV) quantum system,[1] many theoretical and experimental investigations of DV quantum teleportation have been carried out.[5] The original DV quantum teleportation was generalized to continuous variable domain[6,7] using Einstein–Podolsky–Rosen (EPR) entangled state.[8] The first continuous variable (CV) quantum teleportation, quantum teleportation of optical coherent states, was demonstrated experimentally by Furusawa *et al.*[9] using squeezed state entanglement. Experimental demonstration of teleportation of a squeezed thermal state was given.[10] The CV entanglement swapping was implemented.[11,12] The above CV schemes are teleportation of Gaussian state. The CV quantum teleportation using non-Gaussian states of the radiation field as entangled resources was investigated by Anno *et al.*[13] Takei's protocol[10] reported the experimental demonstration of quantum teleportation of a squeezed state, they did not fully discuss the dependence of fidelity on the squeezing parameter in the teleportation of the squeezed coherent state. In this paper, the dependence of fidelity on the squeezing parameter of squeezed coherent states is fully discussed.

The aim of this work is to propose a scheme for the teleportation of squeezed coherent state and to give the analytical expression of fidelity in terms of the squeezing parameter r and quantum channel parameter p. This paper is organized as follows: Section 2 describes the teleportation protocol of the squeezed coherent state. Section 3 gives the fidelity of teleportation as a function of the squeezing parameter r of the teleported squeezed coherent state and the quantum channel parameter p. The conclusions are drawn in Section 4.

2. The teleportation protocol of the single mode

The teleportation protocol of a single mode of the electromagnetic field may be described generally by the following steps (see Fig. 1), the squeezed coherent state can be teleported by this protocol.

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Step 1 Alice prepares the CV EPR pair by combining mode $\hat{a}_5$ with mode $\hat{a}_7$ on beam splitter BS_1, where $\hat{a}_5$ is the 'position' ($X = \frac{1}{2}(\hat{a}+\hat{a}^{\dagger})$) quardrature squeezed state, $\hat{a}_7$ is the 'momentum' ($P = \frac{1}{2i}(\hat{a}-\hat{a}^{\dagger})$) quardrature squeezed state. Then beam $\hat{a}_4$ and beam $\hat{a}_6$ are an EPR pair.

Step 2 Alice keeps the beam $\hat{a}_4$ and sends the beam $\hat{a}_6$ to Bob.

Step 3 Alice combines the unknown squeezed beam $\hat{a}_1$ with beam $\hat{a}_4$ by beam splitter BS_2, producing the beams $\hat{a}_2$ and $\hat{a}_3$, $\hat{a}_1$ is the quantum mode Alice wants to teleport to Bob.

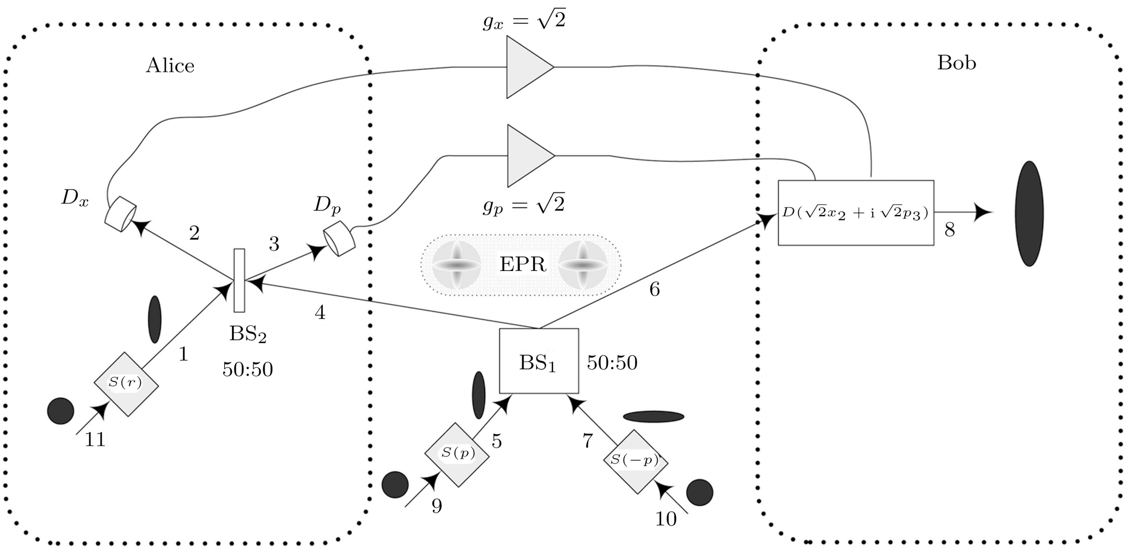

Fig. 1. Schematic representation of teleportation of the squeezed coherent states. The Arab numbers denote the optical modes. The $D(\sqrt{2}x_2 + \mathrm{i}\sqrt{2}p_3)$ denotes the displacement operator. The BS is the optical beam splitter. The $S(r), S(p), S(-p)$ are three squeeze operators, where r is squeezing parameter and p is quantum channel parameter.

Step 4 Alice first measures both the 'position' quadrature X_2 of $\hat{a}_2$ and the 'momentum' quadrature P_3 of $\hat{a}_3$, then she tells the measured result x_2 and p_3 to Bob by the classical communication channel.

Step 5 Bob applies the displacement operator $D(\sqrt{2}x_2 + \mathrm{i}\sqrt{2}p_3)$ on mode $\hat{a}_6$ after he received Alice's measured results. The produced mode $\hat{a}_8$ is the quantum mode Alice wants to teleport to Bob. The mode $\hat{a}_1$ remains in the squeezed coherent state.

3. Teleportation fidelity

In order to evaluate the transmission performance of the proposed protocol, the fidelity F between the teleported mode $\hat{a}_8$ and the original mode $\hat{a}_1$ will be calculated in this section. The definition and the calculation formula of fidelity between two single Gaussian states are first given.

Definition The quantum fidelity was first defined by Jozsa, based on Uhlmann's transition probability. Given two quantum states ρ_1 and ρ_2, we have[14–17]

$$F = \text{Fidelity}(\rho_1, \rho_2) = \left[\text{Tr}\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}\right]^2 = \left\{\sqrt{\frac{2}{\sqrt{\varDelta+\delta}-\sqrt{\delta}}}\exp\left[-\boldsymbol{\beta}^{\mathrm{T}}(\boldsymbol{\varGamma}_1+\boldsymbol{\varGamma}_2)^{-1}\boldsymbol{\beta}\right]\right\}^2, \tag{1}$$

where

$$\hat{X} = \frac{1}{2}(\hat{a}+\hat{a}^{\dagger}),$$
$$\hat{P} = \frac{1}{2\,\mathrm{i}}(\hat{a}-\hat{a}^{\dagger}).$$
$$\hat{R} = (\hat{R}_1, \hat{R}_2) = (\hat{X}, \hat{P}),$$
$$\varDelta = \text{Det}(\boldsymbol{\varGamma}_1+\boldsymbol{\varGamma}_2),$$
$$\delta = (\text{Det}\boldsymbol{\varGamma}_1 - 1)(\text{Det}\boldsymbol{\varGamma}_2 - 1),$$
$$\boldsymbol{\beta} = \boldsymbol{\alpha}_2 - \boldsymbol{\alpha}_1,$$
$$\varGamma_{ij} = 2\langle\Delta\hat{R}_i\Delta\hat{R}_j + \Delta\hat{R}_j\Delta\hat{R}_i\rangle,$$

where $\boldsymbol{\varGamma}_1$ and $\boldsymbol{\varGamma}_2$ are respectively the covariance matrices of the modes $\hat{a}_1$ and $\hat{a}_2$, $\boldsymbol{\alpha}_i$ ($i = 1, 2$) are the mean amplitudes.

For our teleportation protocol, the input mode is $\hat{a}_1$, the output mode is $\hat{a}_8$, so we must calculate the covariance matrices $\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2$ and the mean values $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2$ of $\hat{a}_1$ and $\hat{a}_8$ in order to obtain the fidelity.

In order to obtain the covariance matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_8$, we first calculate the relationship between modes. The quadratures of $\hat{a}_1$, $\hat{a}_5$ and $\hat{a}_7$ are given by the following equations:

$$X_5 = \mathrm{e}^{-p} X_9^{(0)}, \quad P_5 = \mathrm{e}^{p} P_9^{(0)}, \tag{2}$$

$$X_7 = \mathrm{e}^{p} X_{10}^{(0)}, \quad P_7 = \mathrm{e}^{-p} P_{10}^{(0)}, \tag{3}$$

$$X_1 = \mathrm{e}^{-r} X_{11}^{(0)}, \quad P_1 = \mathrm{e}^{r} P_{11}^{(0)}, \tag{4}$$

where

$$\langle X_i^{(0)} \rangle = \langle P_i^{(0)} \rangle = 0,$$
$$\langle \Delta X_i^{(0)} \rangle^2 = \langle \Delta P_i^{(0)} \rangle^2 = \frac{1}{4},\ i = 9, 10,$$

and

$$\langle X_{11}^{(0)} \rangle = x_{\mathrm{in}},\ \langle P_{11}^{(0)} \rangle = p_{\mathrm{in}},$$
$$\langle \Delta X_{11}^{(0)} \rangle^2 = \langle \Delta P_{11}^{(0)} \rangle^2 = \frac{1}{4},$$

i.e., $|\psi_9^{(0)}\rangle$, $|\psi_{10}^{(0)}\rangle$ are vacuum states, $|\psi_{11}^{(0)}\rangle$ is coherent state, p, r are squeezing parameters.

The $\hat{a}_5$ and $\hat{a}_7$ are the input modes of BS_1 (50:50), the output modes $\hat{a}_4$ and $\hat{a}_6$ are given by the following equations:

$$X_4 = \frac{1}{\sqrt{2}}(X_5 + X_7),\ P_4 = \frac{1}{\sqrt{2}}(P_5 + P_7), \tag{5}$$

$$X_6 = \frac{1}{\sqrt{2}}(X_5 - X_7),\ P_6 = \frac{1}{\sqrt{2}}(P_5 - P_7). \tag{6}$$

The output modes $\hat{a}_2$ and $\hat{a}_3$ are given by the following equations:

$$X_2 = \frac{1}{\sqrt{2}}(X_1 + X_4),\ P_2 = \frac{1}{\sqrt{2}}(P_1 + P_4), \tag{7}$$

$$X_3 = \frac{1}{\sqrt{2}}(X_1 - X_4),\ P_3 = \frac{1}{\sqrt{2}}(P_1 - P_4). \tag{8}$$

Alice can precisely measure the X_2 and P_3 by using the homodyne measurement. Bob transforms $\hat{a}_6$ into $\hat{a}_8$ by applying the displacement operator $D(\sqrt{2}x_2 + \mathrm{i}\sqrt{2}p_3)$ after he received Alice's measured results, $\hat{a}_8$ are given by the following equations:

$$X_8 = X_6 + \sqrt{2}X_2,\ P_8 = P_6 + \sqrt{2}P_3. \tag{9}$$

Combining Eqs. (2)–(9), one easily obtains

$$\begin{aligned} X_8 &= X_1 + (X_4 + X_6) = X_1 + \sqrt{2}X_5 \\ &= X_1 + \sqrt{2}\mathrm{e}^{-p} X_9^{(0)}, \\ P_8 &= P_1 + (P_6 - P_4) = P_1 - \sqrt{2}P_7 \\ &= P_1 - \sqrt{2}\mathrm{e}^{-p} P_{10}^{(0)}. \end{aligned} \tag{10}$$

Obviously, $\lim_{p\to\infty} X_8 = X_1$, $\lim_{p\to\infty} P_8 = P_1$, this case corresponds to ideal quantum teleportation.

According to Eq. (4) and the definition of covariance matrix, one easily calculates the covariance matrix and the mean values of the original mode $\hat{a}_1$,

$$\boldsymbol{\Gamma}_1 = \begin{pmatrix} e^{-2r} & 0 \\ 0 & e^{2r} \end{pmatrix}, \tag{11}$$

$$\boldsymbol{\alpha}_1 = \begin{pmatrix} x_{\mathrm{in}} \\ p_{\mathrm{in}} \end{pmatrix}. \tag{12}$$

According to Eq. (10) and the definition of the covariance matrix, the covariance matrix and the mean values of the teleported mode $\hat{a}_8$ are respectively expressed in a similar way as

$$\boldsymbol{\Gamma}_8 = \begin{pmatrix} \mathrm{e}^{-2r} + 2\mathrm{e}^{-2p} & 0 \\ 0 & \mathrm{e}^{2r} + 2\mathrm{e}^{-2p} \end{pmatrix}, \tag{13}$$

$$\boldsymbol{\alpha}_8 = \begin{pmatrix} x_{\mathrm{in}} \\ p_{\mathrm{in}} \end{pmatrix}. \tag{14}$$

Thus

$$\boldsymbol{\beta} = \boldsymbol{\alpha}_8 - \boldsymbol{\alpha}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{15}$$

Substituting Eqs. (11), (13) and (15) into Eq. (1), we obtain the fidelity in terms of the squeezed parameters r and the quantum channel parameter p as

$$F = \frac{1}{\sqrt{1 + \mathrm{e}^{-2p-2r} + \mathrm{e}^{2r-2p} + \mathrm{e}^{-4p}}}. \tag{16}$$

Obviously, for $r > 0$, $\dfrac{\partial f}{\partial r} = \mathrm{e}^{-2p}(\mathrm{e}^{-2r} - \mathrm{e}^{2r})(1 + \mathrm{e}^{-2r-2p} + \mathrm{e}^{2r-2p} + \mathrm{e}^{-4p}) < 0$, i.e., the coherent state is the best quantum signal for quantum teleportation once the quantum channel is built.

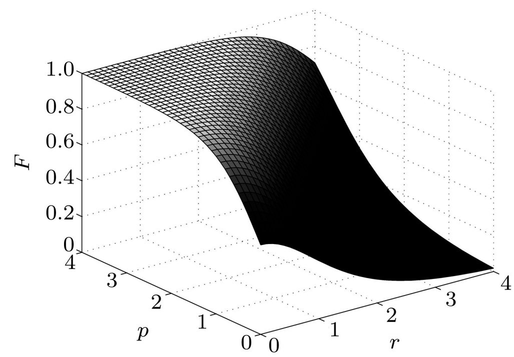

Fig. 2. Teleportation fidelity of the squeezed coherent state in terms of p and r.

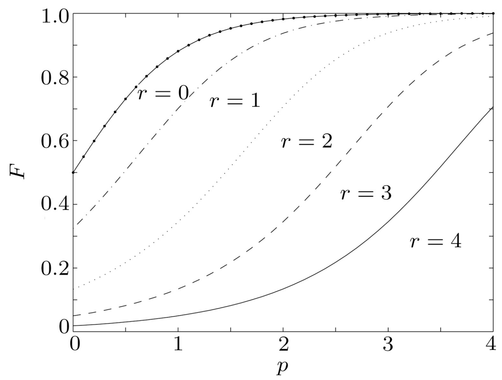

Fig. 3. The relationship of fidelity with quantum channel parameter p given by $r = 0, 1, 2, 3, 4$.

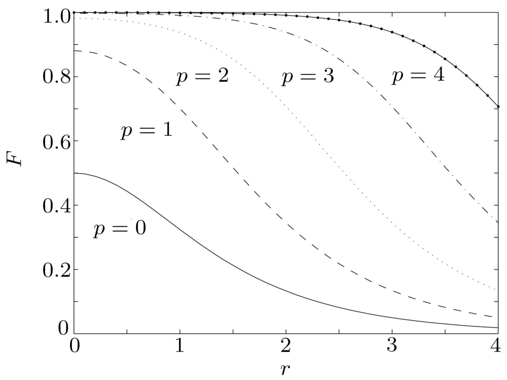

Fig. 4. The relationship of fidelity with the squeezing parameter r given by $p = 0, 1, 2, 3, 4$.

The fidelity of the proposed protocol is plotted in Fig. 2. This figure demonstrates the dependence of fidelity on both p and r. Figure 3 shows that the fidelity increases with the increase of the quantum channel parameter p. The more perfect the quantum channel is, the higher the fidelity of the teleported quantum signal is. In order to improve the performance of quantum teleportation, one can prepare the better CV EPR pair or distill the better EPR pair from more ones by the entanglement purification process.

For $r = 0$, i.e., the quantum state Alice wants to teleport is the coherent state, $F = 0.5$ when $p = 0$. This corresponds to the classical telportation and the result is compatible with the result.[9] A fidelity higher than 0.5 is not possible for coherent states without the use of entanglement.

Figure 4 shows that the fidelity decreases with the increase of the squeezing parameter r of the squeezed coherent state. As we can see, the coherent state $r = 0$ has the highest fidelity once the quantum channel is built, i.e., the squeezing parameter of two-mode squeezed state p equals a special value. Thus the coherent state is the best quantum signal for quantum teleportation after the quantum channel is built.

4. Conclusion

We have investigated the teleportation protocol of a single mode of electromagnetic field. This teleportation protocol can teleport the squeezed coherent state. The analytical relationship of the fidelity with both the squeezing parameter r and the quantum channel parameter p is given. The results of analysis show that the fidelity increases with the increase of the quantum channel parameter p, while the fidelity decreases with the increase of the squeezing parameter r of the squeezed coherent state. That is to say, the coherent state is the best quantum signal for quantum teleportation once quantum channel is built.

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