The multiparty coherent channel and its implementation with linear optics

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Abstract: The continuous-variable coherent (conat) channel is a useful resource for coherent communication, supporting coherent teleportation and coherent superdense coding. We extend the conat channel to multiparty conditions by proposing definitions on multiparty position-quadrature and momentum-quadrature conat channel. We additionally provide two methods to implement this channel using linear optics. One method is the multiparty version of coherent communication assisted by entanglement and classical communication (CCAECC). The other is multiparty coherent superdense coding.

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OCIS codes: (060.5565) Quantum communications; (270.5565) Quantum communications.

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1. Introduction

The coherent bit (cobit) with discrete variables (DV) proposed by Aram Harrow [1] is a powerful resource intermediate between a quantum bit (qubit) and a classical bit (cbit). According to [1], a qubit channel can be described as: $|x\rangle^A \to |x\rangle^B$ ($\{|x\rangle\}_{x\in\{0,1\}}$ is a basis for $\mathbb{C}^2$. A is sender Alice, and B is receiver Bob). A cbit channel is: $|x\rangle^A \to |x\rangle^B|x\rangle^E$ (E is inaccessible environment).

#190588 - $15.00 USD Received 16 May 2013; revised 22 Jun 2013; accepted 24 Jun 2013; published 15 Aug 2013
(C) 2013 OSA 26 August 2013 | Vol. 21, No. 17 | DOI:10.1364/OE.21.019790 | OPTICS EXPRESS 19790

A bipartite cobit channel with DV can be described by the isometry: $|x\rangle^A \rightarrow |x\rangle^A|x\rangle^B$. For example, if Alice possesses an arbitrary qubit: $|\psi\rangle^A = \alpha|0\rangle^A + \beta|1\rangle^A$ and transmits it through a cobit channel, the channel generates: $|\phi\rangle^{AB} = \alpha|0\rangle^A|0\rangle^B + \beta|1\rangle^A|1\rangle^B$. The process maintains the coherent superposition property of Alice's original state, this is the reason for the channel's name[1]. In [2], Wilde and Brun compared and pointed out the differences and connections among concepts about classical communication, quantum communication, entanglement and cobit channel.

Recently, Wilde, Krovi, and Brun extended cobit channel concepts from discrete variables to continuous variables (CV), and they introduced the notion of 'conat channel'[3] as the CV counterpart of the DV 'cobit channel'. In their work, Wilde and Brun proposed definitions of ideal position-quadrature (PQ) and momentum-qudrature (MQ) conat channel in Schroedinger-picture: $|x\rangle^A \rightarrow |x\rangle^A|x\rangle^B$ and $|p\rangle^A \rightarrow |p\rangle^A|p\rangle^B$ respectively, where $|x\rangle$ and $|p\rangle$ represents position and momentum eigenstate. Nonideal (finitely squeezed) conat channels are also discussed in the Heisenberg picture.

The coherent channel provides for coherent communication. Coherent communication has several useful characteristics and applications. It provides a coherent version of continuous-variable teleportation and continuous-variable superdense coding, which are proved dual under resource reversal [1, 4]. In addition, it can be applied in remote state preparation (RSP), consuming fewer entangled states than standard ways [1]. Moreover, coherent communication also proves useful in error correcting codes [5, 6].

In this paper, we extend the notion of conat channel to multiparty situations in order to perform multiparty coherent communication. Additionally, we propose two implementations using linear optical techniques and analyze the noise accumulation in different EPR [7] resources. Multiparty conat channel inherits all the useful properties from bipartite conat channel.

Our paper is organized as follows. In Section 2, we provide a general definition of multiparty position-quadrature (PQ) and momentum-qudrature (MQ) conat channel in Heisenberg picture. In Section 3, two implementations of the protocol with linear optical devices are outlined. A brief conclusion is given in Section 4.

2. Definitions of multiparty conat channel

We propose a general definition of multiparty position-quadrature (PQ) and momentum-qudrature (MQ) conat channel in Heisenberg picture.

The channel has only one sender (Alice) and n receivers (Alice, Bob, Claire $\cdots$ and Nick). Notice that the sender is also among the receivers. We denote sender Alice as A and the received massages through the channel as $A', B', C', \cdots, N'$. The multiparty PQ conat channel $\tilde{\Delta}_X$ copies the position quadrature of the sender to all the receivers with noise. The resulting multimode state is similar to multiparty Greenberger-Horne-Zeilinger (GHZ) entangled states[8]. The difference is that total momentum of the output modes is close to the original momentum $\hat{p}_A$, encoding messages into all the receivers involved, while the total momentum of GHZ entangled state is zero. Multiparty MQ conat channel is similarly defined.

Definition 1: A multiparty PQ conat channel $\tilde{\Delta}_X$.

Mapping:

$$[\hat{x}_A \ \hat{p}_A]^T \rightarrow [\hat{x}_{A'} \ \hat{p}_{A'} \ \hat{x}_{B'} \ \hat{p}_{B'} \ \cdots \ \hat{x}_{N'} \ \hat{p}_{N'}]^T \tag{1}$$

#190588 - $15.00 USD Received 16 May 2013; revised 22 Jun 2013; accepted 24 Jun 2013; published 15 Aug 2013
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Constraints:

$$\begin{cases} \hat{x}_{A'} = \hat{x}_A \\ \hat{x}_{B'} = \hat{x}_A + \hat{x}_{\Delta_X}^1 \\ \hat{x}_{C'} = \hat{x}_A + \hat{x}_{\Delta_X}^2 \\ \vdots \\ \hat{x}_{N'} = \hat{x}_A + \hat{x}_{\Delta_X}^{n-1} \\ \hat{p}_{A'} = \hat{p}_A + \hat{p}_{\Delta_X} \end{cases} \tag{2}$$

$$\begin{cases} \langle \hat{x}_{\Delta_X}^1 \rangle = \langle \hat{x}_{\Delta_X}^2 \rangle = \cdots = \langle \hat{x}_{\Delta_X}^{n-1} \rangle = 0 \\ \langle \hat{p}_{A'} + \hat{p}_{B'} + \hat{p}_{C'} + \cdots + \hat{p}_{N'} \rangle = \langle \hat{p}_A \rangle \Leftrightarrow \langle \hat{p}_{\Delta_X} + \hat{p}_{B'} + \hat{p}_{C'} + \cdots + \hat{p}_{N'} \rangle = 0 \\ \langle (\hat{x}_{\Delta_X}^1)^2 \rangle \le \varepsilon_1 \\ \langle (\hat{x}_{\Delta_X}^2)^2 \rangle \le \varepsilon_2 \\ \vdots \\ \langle (\hat{x}_{\Delta_X}^{n-1})^2 \rangle \le \varepsilon_{n-1} \\ \langle (\hat{p}_{\Delta_X} + \hat{p}_{B'} + \hat{p}_{C'} + \cdots + \hat{p}_{N'})^2 \rangle \le \varepsilon_n \end{cases} \tag{3}$$

The canonical commutation relations of the Heisenberg-picture observables are as follows:

$$[\hat{x}_{A'}, \hat{p}_{A'}] = [\hat{x}_{B'}, \hat{p}_{B'}] = \cdots = [\hat{x}_{N'}, \hat{p}_{N'}] = i \tag{4}$$

The constraints indicate that: Alice's position quadrature remains unchanged, and it is copied to all the other receivers with the additional noise $\hat{x}_{\Delta_X}^1$, $\hat{x}_{\Delta_X}^2$ $\cdots$ $\hat{x}_{\Delta_X}^{n-1}$ respectively; the total momentum of the multimode state is close to Alice's original momentum with a noise $\hat{p}_{\Delta_X}$. The parameters ε_1, $\varepsilon_2 \cdots$ and ε_n in (3) describe the performance of channel in the meaning of noise.

Definition 2: A multiparty MQ conat channel $\tilde{\Delta}_P$.

Mapping:

$$[\hat{x}_A \ \hat{p}_A]^T \to [\hat{x}_{A'} \ \hat{p}_{A'} \ \hat{x}_{B'} \ \hat{p}_{B'} \ \cdots \ \hat{x}_{N'} \ \hat{p}_{N'}]^T \tag{5}$$

Constraints:

$$\begin{cases} \hat{p}_{A'} = \hat{p}_A \\ \hat{p}_{B'} = \hat{p}_A + \hat{p}_{\Delta_P}^1 \\ \hat{p}_{C'} = \hat{p}_A + \hat{p}_{\Delta_P}^2 \\ \vdots \\ \hat{p}_{N'} = \hat{p}_A + \hat{p}_{\Delta_P}^{n-1} \\ \hat{x}_{A'} = \hat{x}_A + \hat{x}_{\Delta_P} \end{cases} \tag{6}$$

$$\begin{cases} \langle \hat{p}_{\Delta_P}^1 \rangle = \langle \hat{p}_{\Delta_P}^2 \rangle = \cdots = \langle \hat{p}_{\Delta_P}^{n-1} \rangle = 0 \\ \langle \hat{x}_{A'} + \hat{x}_{B'} + \hat{x}_{C'} + \cdots + \hat{x}_{N'} \rangle = \langle \hat{x}_A \rangle \Leftrightarrow \langle \hat{x}_{\Delta_P} + \hat{x}_{B'} + \hat{x}_{C'} + \cdots + \hat{x}_{N'} \rangle = 0 \\ \langle (\hat{p}_{\Delta_P}^1)^2 \rangle \le \varepsilon_1 \\ \langle (\hat{p}_{\Delta_P}^2)^2 \rangle \le \varepsilon_2 \\ \vdots \\ \langle (\hat{p}_{\Delta_P}^{n-1})^2 \rangle \le \varepsilon_{n-1} \\ \langle (\hat{x}_{\Delta_P} + \hat{x}_{B'} + \hat{x}_{C'} + \cdots + \hat{x}_{N'})^2 \rangle \le \varepsilon_n \end{cases} \tag{7}$$

The canonical commutation relations of the Heisenberg-picture observables are as follows:

$$[\hat{x}_{A'}, \hat{p}_{A'}] = [\hat{x}_{B'}, \hat{p}_{B'}] = \cdots = [\hat{x}_{N'}, \hat{p}_{N'}] = i \tag{8}$$

#190588 - $15.00 USD Received 16 May 2013; revised 22 Jun 2013; accepted 24 Jun 2013; published 15 Aug 2013
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3. Implementations of multiparty conat channel using linear optics

In this section, we outline two methods to implement multiparty conat channel. The first is multiparty coherent communication assisted by entanglement and classical communication (CCAECC) [2]. The second is multiparty superdense coding, which can implements multiparty PQ and MQ coherent channels simultaneously.

Method 1:

This method requires $(n+1)$-party GHZ entangled state and classical communication channel, as shown in Fig. 1. For simplicity, we implement a three-party PQ conat channel as a demonstration. We will generalize to n-party PQ conat channel later (multiparty MQ conat channel is similar).

In this method, Alice wants to transmit her mode A. Alice, Bob and Claire share a four-mode entangled state A_1, A_2, B and C. A_1 and A_2 belong to Alice, B belongs to Bob and C belongs to Claire.

We use Loock and Braunstein's protocol [9] to generate a four-mode GHZ entanglement. This protocol starts from 4 original vacuum states ($\hat{x}_1^{(0)}$, $\hat{p}_1^{(0)}$, $\hat{x}_2^{(0)}$, $\hat{p}_2^{(0)}$, $\hat{x}_3^{(0)}$, $\hat{p}_3^{(0)}$, $\hat{x}_4^{(0)}$ and $\hat{p}_4^{(0)}$), squeezes them with squeezing coefficients r_1, r_2, r_3, r_4, and generates entangled states A_1, A_2, B and C. The equations with coefficients calculated are given:

$$\begin{cases} \hat{x}_{A_1} &= \frac{1}{\sqrt{4}}e^{+r_1}\hat{x}_1^{(0)} + \sqrt{\frac{3}{4}}e^{-r_2}\hat{x}_2^{(0)} \\ \hat{p}_{A_1} &= \frac{1}{\sqrt{4}}e^{-r_1}\hat{p}_1^{(0)} + \sqrt{\frac{3}{4}}e^{+r_2}\hat{p}_2^{(0)} \\ \hat{x}_{A_2} &= \frac{1}{\sqrt{4}}e^{+r_1}\hat{x}_1^{(0)} - \frac{1}{\sqrt{12}}e^{-r_2}\hat{x}_2^{(0)} + \sqrt{\frac{2}{3}}e^{-r_3}\hat{x}_3^{(0)} \\ \hat{p}_{A_2} &= \frac{1}{\sqrt{4}}e^{-r_1}\hat{p}_1^{(0)} - \frac{1}{\sqrt{12}}e^{+r_2}\hat{p}_2^{(0)} + \sqrt{\frac{2}{3}}e^{+r_3}\hat{p}_3^{(0)} \\ \hat{x}_B &= \frac{1}{\sqrt{4}}e^{+r_1}\hat{x}_1^{(0)} - \frac{1}{\sqrt{12}}e^{-r_2}\hat{x}_2^{(0)} - \frac{1}{\sqrt{6}}e^{-r_3}\hat{x}_3^{(0)} + \frac{1}{\sqrt{2}}e^{-r_4}\hat{x}_4^{(0)} \\ \hat{p}_B &= \frac{1}{\sqrt{4}}e^{-r_1}\hat{p}_1^{(0)} - \frac{1}{\sqrt{12}}e^{+r_2}\hat{p}_2^{(0)} - \frac{1}{\sqrt{6}}e^{+r_3}\hat{p}_3^{(0)} + \frac{1}{\sqrt{2}}e^{+r_4}\hat{p}_4^{(0)} \\ \hat{x}_C &= \frac{1}{\sqrt{4}}e^{+r_1}\hat{x}_1^{(0)} - \frac{1}{\sqrt{12}}e^{-r_2}\hat{x}_2^{(0)} - \frac{1}{\sqrt{6}}e^{-r_3}\hat{x}_3^{(0)} - \frac{1}{\sqrt{2}}e^{-r_4}\hat{x}_4^{(0)} \\ \hat{p}_C &= \frac{1}{\sqrt{4}}e^{-r_1}\hat{p}_1^{(0)} - \frac{1}{\sqrt{12}}e^{+r_2}\hat{p}_2^{(0)} - \frac{1}{\sqrt{6}}e^{+r_3}\hat{p}_3^{(0)} - \frac{1}{\sqrt{2}}e^{+r_4}\hat{p}_4^{(0)} \end{cases} \tag{9}$$

And we assume that all squeezing coefficients equal to r.

Then we present the transformations of the operators in Heisenberg picture as follows:

Step 1. Alice mixes mode A and A_1 locally on a balanced (50%) beam splitter (BS), generating modes (+) and (-).

$$\begin{cases} \hat{x}_+ = (\hat{x}_A + \hat{x}_{A_1})/\sqrt{2}\ ,\ \hat{p}_+ = (\hat{p}_A + \hat{p}_{A_1})/\sqrt{2} \\ \hat{x}_- = (\hat{x}_A - \hat{x}_{A_1})/\sqrt{2}\ ,\ \hat{p}_- = (\hat{p}_A - \hat{p}_{A_1})/\sqrt{2} \end{cases} \tag{10}$$

Step 2. We express $\hat{x}_{A_2}$, $\hat{p}_{A_2}$, $\hat{x}_B$ and $\hat{x}_C$ in terms of $\hat{x}_-$ and $\hat{p}_+$.

$$\begin{cases} \hat{x}_{A_2} &= \hat{x}_A - \hat{x}_{A_1} + \hat{x}_{A_2} - \sqrt{2}\hat{x}_- \\ \hat{p}_{A_2} &= \hat{p}_A + (\hat{p}_{A_1} + \hat{p}_{A_2} + \hat{p}_B + \hat{p}_C) - \hat{p}_B - \hat{p}_C - \sqrt{2}\hat{p}_+ \\ \hat{x}_B &= \hat{x}_A - \hat{x}_{A_1} + \hat{x}_B - \sqrt{2}\hat{x}_- \\ \hat{x}_C &= \hat{x}_A - \hat{x}_{A_1} + \hat{x}_C - \sqrt{2}\hat{x}_- \end{cases} \tag{11}$$

Then Alice measures $\hat{x}_-$ and $\hat{p}_+$ using homodyne detection, and operator $\hat{x}_-$, $\hat{p}_+$ collapse to value x_-, p_+. Then she sends the value x_- to Bob and Claire over a classical communication channel. Suppose the photodetectors have efficiency η.

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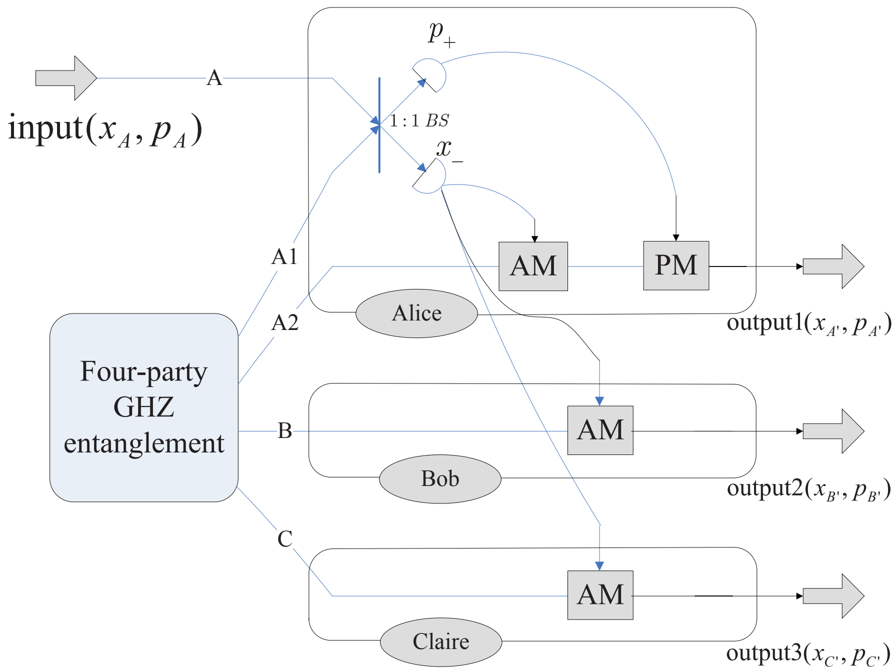

Fig. 1. The three-party position-quadrature (PQ) protocol requires a four-mode GHZ entangled state, and we label the four modes as A_1, A_2, B, C. Alice possesses mode A_1 and A_2, Bob possesses mode B and Claire possesses mode C. Alice has an input mode A to be transmitted. The elements in this figure: *AM* is an amplitude modulator which displaces the position quadrature of an optical mode [2]. *PM* is a phase modulator which kicks the momentum quadrature of an optical mode [2]. *BS* is a beam splitter.

Step 3. Alice displaces the position quadrature of her mode A_2 by $\sqrt{2}x_-$ and her momentum quadrature by $\sqrt{2}p_+$, Bob and Claire both displace the position quadrature of their mode B and C by $\sqrt{2}x_-$, resulting mode A', B' and C':

$$\begin{cases} \hat{x}_{A'} &= \hat{x}_A - (\hat{x}_{A_1} - \hat{x}_{A_2}) - \sqrt{2(1-\eta)/\eta}\,\hat{x}_1^{(0)} \\ \hat{p}_{A'} &= \hat{p}_A + (\hat{p}_{A_1} + \hat{p}_{A_2} + \hat{p}_B + \hat{p}_C) - \hat{p}_B - \hat{p}_C + \sqrt{2(1-\eta)/\eta}\,\hat{p}_2^{(0)} \\ \hat{x}_{B'} &= \hat{x}_A - (\hat{x}_{A_1} - \hat{x}_B) - \sqrt{2(1-\eta)/\eta}\,\hat{x}_1^{(0)} \\ \hat{p}_{B'} &= \hat{p}_B \\ \hat{x}_{C'} &= \hat{x}_A - (\hat{x}_{A_1} - \hat{x}_C) - \sqrt{2(1-\eta)/\eta}\,\hat{x}_1^{(0)} \\ \hat{p}_{C'} &= \hat{p}_C \end{cases} \tag{12}$$

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Finally, we can find that the results satisfy the constraints in (2) and (3):

$$\begin{cases} \langle \hat{x}_{B'} - \hat{x}_{A'}) \rangle = \langle \hat{x}_B - \hat{x}_{A_2} \rangle = 0 \\ \langle (\hat{x}_{B'} - \hat{x}_{A'})^2 \rangle = \langle (\hat{x}_B - \hat{x}_{A_2})^2 \rangle = 2e^{-2r} \\ \langle \hat{x}_{C'} - \hat{x}_{A'} \rangle = \langle \hat{x}_C - \hat{x}_{A_2} \rangle = 0 \\ \langle (\hat{x}_{C'} - \hat{x}_{A'})^2 \rangle = \langle (\hat{x}_C - \hat{x}_{A_2})^2 \rangle = 2e^{-2r} \\ \langle \hat{p}_{\Delta_X} + \hat{p}_{B'} + \hat{p}_{C'} \rangle = \langle \hat{p}_{A_1} + \hat{p}_{A_2} + \hat{p}_B + \hat{p}_C + \sqrt{2(1-\eta)/\eta}\,\hat{p}_2^{(0)} \rangle = 0 \\ \langle (\hat{p}_{\Delta_X} + \hat{p}_{B'} + \hat{p}_{C'})^2 \rangle = 4e^{-2r} + 2(1-\eta)/\eta \end{cases} \tag{13}$$

Therefore, the above operations implement a $[4e^{-2r} + 2(1-\eta)/\eta]$-approximate PQ coherent channel[2]. In this example , n is equal to 3. As n increases, we can easily calculate that: ε_1, $\varepsilon_2 \cdots \varepsilon_{n-1}$ remain unchanged (=$2e^{-2r}$) and ε_n amounts to $(n+1)e^{-2r} + 2(1-\eta)/\eta$. When it comes to the general n-party conditions, we can implement an $[(n+1)e^{-2r} + 2(1-\eta)/\eta]$-approximate coherent channel similarly by using the $(n+1)$-party GHZ entanglement.

The implementation of multiparty MQ conat channel is similar to multiparty PQ conat channel by exchanging the operator $\hat{x}$ and $\hat{p}$. The required resource is a GHZ-like entangled state with total position $\hat{x}_{A_1} + \hat{x}_{A_2} + \hat{x}_B + \cdots + \hat{x}_N \to 0$ and certain momenta equal. So we omit these discussions here.

Method 2:

Widle et al. proposed a protocol of coherent superdense coding recently [3]. Inspired by their work, we provide a multiparty version of coherent superdense coding. The protocol is equivalent to multiparty conat channels: a multiparty PQ conat channel and a multiparty MQ conat channel. In this method, the channel has only one sender which has two modes to transmit, and finally has n receivers obtained the two modes respectively. In addition, $(n-1)$ prepared EPR pairs among the receivers are required.

We also use Loock and Braunstein's method to generate an EPR pair. In Heisenberg picture:

$$\begin{cases} \hat{x}_1 = (e^{+r}\hat{x}_1^{(0)} + e^{-r}\hat{x}_2^{(0)})/\sqrt{2}, \quad \hat{p}_1 = (e^{-r}\hat{p}_1^{(0)} + e^{+r}\hat{p}_2^{(0)})/\sqrt{2} \\ \hat{x}_2 = (e^{+r}\hat{x}_1^{(0)} - e^{-r}\hat{x}_2^{(0)})/\sqrt{2}, \quad \hat{p}_2 = (e^{-r}\hat{p}_1^{(0)} - e^{+r}\hat{p}_2^{(0)})/\sqrt{2} \end{cases} \tag{14}$$

Local quantum nondemolition (QND) measurements[10] are also employed. The interaction Hamiltonian of an ideal quadrature QND measurement has the following form:

$$\hat{H} = \hbar\chi\hat{x}_1\hat{x}_2 \tag{15}$$

where χ is coupling strength. The transformation of an ideal continuous-variable QND interaction with unit gain on two input optical modes, denoted as $\hat{Q}$ operation, is expressed as:

$$\hat{Q}_{1,2} \Leftrightarrow \begin{cases} \hat{x}_{1'} = \hat{x}_1, & \hat{p}_{1'} = \hat{p}_1 - \hat{p}_2 \\ \hat{x}_{2'} = \hat{x}_1 + \hat{x}_2, & \hat{p}_{2'} = \hat{p}_2 \end{cases} \tag{16}$$

Similarly, QND interaction with a phase adjust[10] can be described as $\hat{Q}^{\,p}$ operation:

$$\hat{Q}_{1,2}^{\,p} \Leftrightarrow \begin{cases} \hat{x}_{1'} = \hat{x}_1 - \hat{x}_2, & \hat{p}_{1'} = \hat{p}_1 \\ \hat{x}_{2'} = \hat{x}_2, & \hat{p}_{2'} = \hat{p}_1 + \hat{p}_2 \end{cases} \tag{17}$$

Our protocol requires $(n-1)$ EPR pairs for receivers. We illustrate these requirements in Fig. 2, on the condition that n equals to 3. We use a graph to illustrate the entanglement relations among the parties involved. A node in the graph represents an individual party in the channel,

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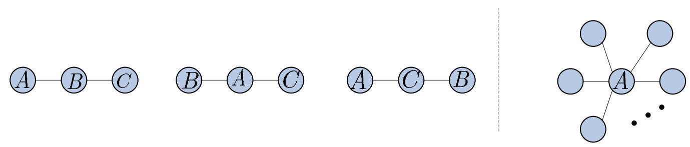

Fig. 2. On the left side, we present all the scenarios of graph which represents the prepared entanglement resources when n is equal to 3. The number of scenarios is three. On the right side, the graph indicates the best prepared entanglement resources for n-party conat channel.

and the edge between two nodes indicates EPR entanglement relation between two parties. We will introduce concepts from graph theory: when two nodes are terminals of a edge, they are called ’adjacent’; we define two nodes as ‘connected’ when a path exists between them; a connected graph is a graph in which any two of the nodes are connected. This method requires that the graph of entanglement resources be a connected graph. The channel has n parties involved. The $(n-1)$ EPR pairs prepared among the n parties ensure that the representing graph is connected, and any two nodes of the graph has only one path.

Without loss of generality, we discuss two scenarios in Fig. 2. Scenario 1 corresponds to the first graph in Fig. 2, Scenario 2 corresponds to the second graph. Since n is equal to 3, the channel involves one sender and three receivers, two EPR pairs is required.

Scenario 1. In this scenario, as shown in Fig. 3. Alice and Bob shares an EPR entanglement pair: mode 3 for Alice and mode 4 for Bob, Bob and Claire share an EPR entanglement pair: mode 5 for Bob and mode 6 for Claire. Alice possesses mode 1 and 2 which are to be transmitted. Fig. 3 gives the schematic linear optics circuit.

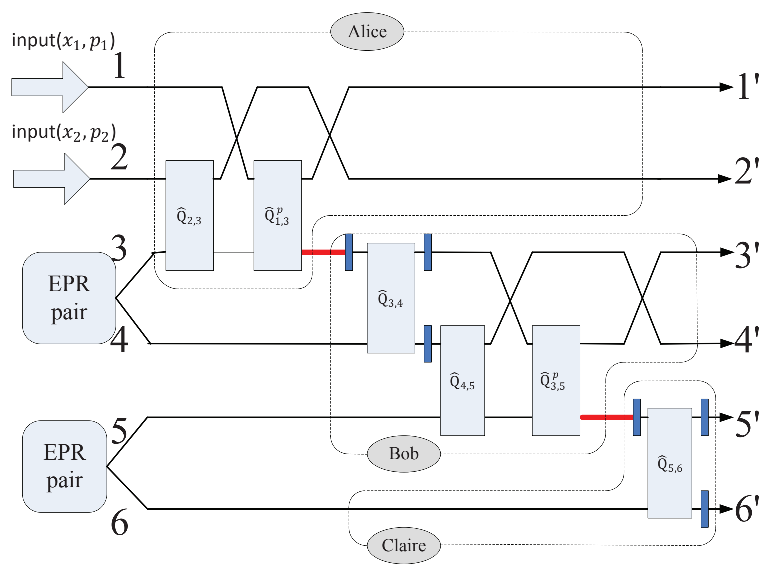

Fig. 3. This figure outlines our scheme. The thick red line represents a quantum channel between two parties. The local operations and modes are enclosed by dashed lines with the name. The blue thin rectangle represents phase shifter at an angle of π.

In step 1, Alice performs $\hat{Q}_{2,3}$, and then $\hat{Q}^p_{1,3}$. In step 2, Alice sends her mode 3 to Bob

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through quantum channel, so Bob now possesses mode 3, 4 and 5. Then he performs a series of QND interactions: first $\hat{Q}_{3,4}$, then $\hat{Q}_{4,5}$, finally $\hat{Q}^{p}_{3,5}$. In step 3, Bob sends mode 5 to Claire through a quantum channel. Claire couples his two modes, $\hat{Q}_{5,6}$. then we can get the resulting modes in Heisenberg picture.

$$\begin{cases} \hat{x}_{1'} = \hat{x}_1 - (\hat{x}_2 + \hat{x}_3) \ , & \hat{p}_{1'} = \hat{p}_1 \\ \hat{x}_{2'} = \hat{x}_2 \ , & \hat{p}_{2'} = \hat{p}_2 - \hat{p}_3 \\ \hat{x}_{3'} = \hat{x}_2 + \hat{x}_3 - (\hat{x}_2 + \hat{x}_3 - \hat{x}_4 + \hat{x}_5) & \hat{p}_{3'} = \hat{p}_1 + \hat{p}_3 + \hat{p}_4 \\ \hat{x}_{4'} = \hat{x}_2 + \hat{x}_3 - \hat{x}_4 \ , & \hat{p}_{4'} = -\hat{p}_4 - \hat{p}_5 \\ \hat{x}_{5'} = \hat{x}_2 + \hat{x}_3 - \hat{x}_4 + \hat{x}_5 \ , & \hat{p}_{5'} = \hat{p}_1 + \hat{p}_3 + \hat{p}_4 + \hat{p}_5 + \hat{p}_6 \\ \hat{x}_{6'} = \hat{x}_2 + \hat{x}_3 - \hat{x}_4 + \hat{x}_5 - \hat{x}_6 \ , & \hat{p}_{6'} = -\hat{p}_6 \end{cases} \tag{18}$$

Modes 1', 3', 5' implement a three-party MQ conat channel by satisfying the constraints in definition 2; and modes 2', 4', 6' satisfy definition 1 and work as a three-party PQ conat channel.

Scenario 2. In this scenario, Alice possesses four modes at the beginning: mode 1, 2, 3, 5, Bob possesses mode 4 and Claire possesses mode 6. Mode 1 and 2 is to be transmitted while modes 3, 4, 5, 6 are the auxiliary modes. Modes 3, 4 are EPR pair, as well as modes 5, 6. We give Fig. 4 to describe this protocol.

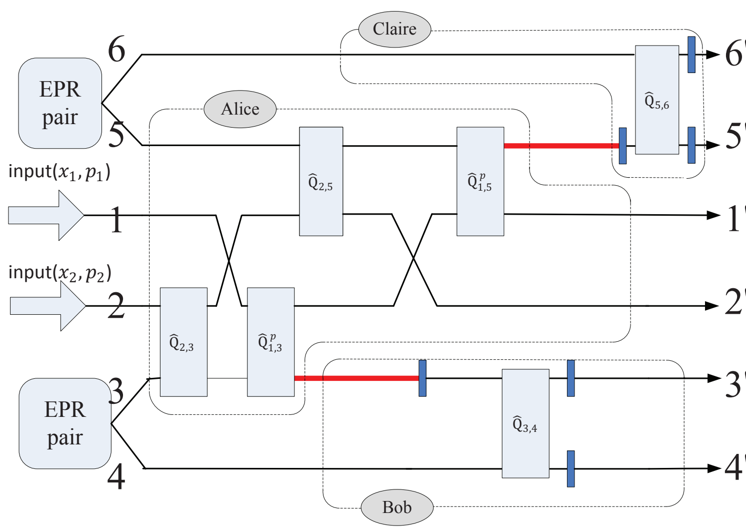

Fig. 4. This figure outlines our scheme. The thick red line represents a quantum channel between two parties. The local operations and modes are enclosed by dashed lines with the name. The blue thin rectangle represents phase shifter at an angle of π.

In step 1, Alice performs a series of QND interactions: first, $\hat{Q}_{2,3}$; then $\hat{Q}^{p}_{1,3}$; then $\hat{Q}_{2,5}$; finally $\hat{Q}^{p}_{1,5}$. In step 2, Alice sends mode 3 to Bob and mode 5 to Claire through quantum channels. Then Bob possesses two modes 3, 4 and Claire possesses modes 5, 6. They perform local QND interactions on their two modes respectively, $\hat{Q}_{5,6}$ and $\hat{Q}_{3,4}$. The resulting modes are given as follows:

#190588 - $15.00 USD Received 16 May 2013; revised 22 Jun 2013; accepted 24 Jun 2013; published 15 Aug 2013
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$$\begin{cases} \hat{x}_{1'} = \hat{x}_1 - (\hat{x}_2 + \hat{x}_3) - (\hat{x}_2 + \hat{x}_5)\ , & \hat{p}_{1'} = \hat{p}_1 \\ \hat{x}_{2'} = \hat{x}_2\ , & \hat{p}_{2'} = \hat{p}_2 - \hat{p}_3 - \hat{p}_5 \\ \hat{x}_{3'} = \hat{x}_2 + \hat{x}_3\ , & \hat{p}_{3'} = \hat{p}_1 + \hat{p}_3 + \hat{p}_4 \\ \hat{x}_{4'} = \hat{x}_2 + \hat{x}_3 - \hat{x}_4\ , & \hat{p}_{4'} = -\hat{p}_4 \\ \hat{x}_{5'} = \hat{x}_2 + \hat{x}_5\ , & \hat{p}_{5'} = \hat{p}_1 + \hat{p}_5 + \hat{p}_6 \\ \hat{x}_{6'} = \hat{x}_2 + \hat{x}_5 - \hat{x}_6\ , & \hat{p}_{6'} = -\hat{p}_6 \end{cases} \tag{19}$$

As in scenario 1, the output modes 1', 3', 5' perform a three-party MQ conat channel and modes 2', 4', 6' perform a three-party PQ conat channel. So two multiparty conat channels are produced in this protocol.

Then we discuss the noise of the channels in these two scenarios, we assume that the local QND interaction is ideal. So we get that

In Scenario 1:

$$\begin{cases} \textit{PQ conat channel} & : \ \varepsilon_1 = 2e^{-2r},\ \varepsilon_2 = 4e^{-2r},\ \varepsilon_3 = 4e^{-2r} \\ \textit{MQ conat channel} & : \ \varepsilon_1 = 2e^{-2r},\ \varepsilon_2 = 4e^{-2r},\ \varepsilon_3 = 0 \end{cases} \tag{20}$$

In Scenario 2:

$$\begin{cases} \textit{PQ conat channel} & : \ \varepsilon_1 = 2e^{-2r},\ \varepsilon_2 = 2e^{-2r},\ \varepsilon_3 = 4e^{-2r} \\ \textit{MQ conat channel} & : \ \varepsilon_1 = 2e^{-2r},\ \varepsilon_2 = 2e^{-2r},\ \varepsilon_3 = 0 \end{cases} \tag{21}$$

Comparing these two scenarios, we find that the noise of scenario 2 is lower. We can reach a conclusion that the longer the path between one party and Alice, the larger the noise of the party and of the channel. As shown in Fig. 2, the first graph represents Scenario 1, the second graph represents Scenario 2. For instance, in Scenario 1, the length of the path between Bob and Alice is one, and the length of the path between Claire and Alice is two. So Claire gets larger noise $\varepsilon_2 = 4e^{-2r}$. The longer the path between one party and Alice, the larger the accumulation of the noise this party gets. In Scenario 2, the length of the path between Alice and any party is one, there is no accumulation of the noise, so Scenario 2 is better. For n-party conat channel, the best prepared entanglement resources is the rightmost graph in Fig. 2.

4. Conclusion

Coherent bits (cobits) are intermediate in power between qubits and cbits. Qubit sources can be used to simulate cobit sources, and cobit sources can simulate cbit sources[1]. Coherent communications offers a new view of quantum information elements.

In this paper, we extended the notion of continuous-variable coherent (conat) channel to multiparty conditions and proposed two definitions of it. Then we propose two implementations of multiparty conat channel using linear optics. One method is the multiparty version of coherent communication assisted by entanglement and classical communication (CCAECC). The other is multiparty coherent superdense coding which implements two multiparty coherent channels. We also discuss the noise of the channel in two scenarios when n equals to 3.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grants No. 61102053), the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry, SMC Excellent Young Faculty program (2011), SJTU Young Teacher Foundation (Grants No. A2831B) and SJTU PRP (Grants No. T03013002).